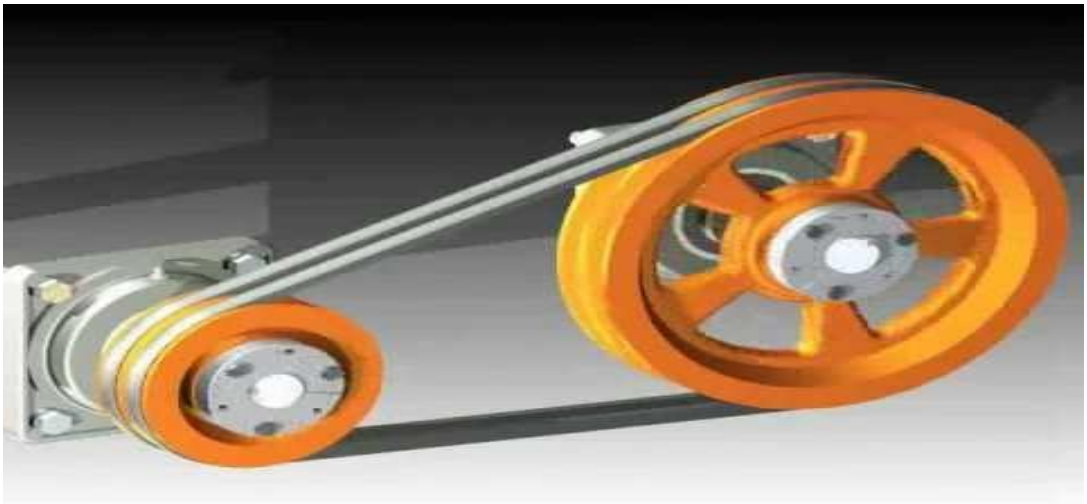
GOVT. POLYTECHNIC, BHADRAK

Theory of machine (Th-01)

(As per the2020-21syllabus of the SCTE&VT, Bhubaneswar, Odisha)



Fourth Semester

Mechanical Engg.

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THEORY OF MACHINES (Th-01)

Chapter No.	Topics	Periods as perSyllabus	Required period	Expected Marks	
01	Simple Mechanism	08	10	07	20
02	Friction	12	14	06	10
03	Power Transmission	12	14	07	10
04	Governor& Flywheel	12	14		20
05	Balancingof Machines	08	09		10
06	Vibration of MachineParts	08	09		10
TOTAL		**60**	**70**	**20**	**80**

SYLLABUS

1.0 Simple mechanism 1.1 Link ,kinematic chain, mechanism, machine 1.2 Inversion, fourbarlinkmechanismanditsinversion1.3Lowerpairandhigherpair1.4Cam and followers

2.0Friction2.1Frictionbetweennutandscrewforsquarethread,screwjack2.2 Bearing and its classification, Description of roller, needle roller& ball bearings. 2.3 Torque transmission in flat pivot& conical pivot bearings. 2.4 Flat collar bearing of singleandmultipletypes.2.5Torquetransmissionforsingleandmultipleclutches
2.6 Working of simple frictional brakes.2.7 Working of Absorption type ofdynamometer

3.0PowerTransmission3.1Conceptofpowertransmission3.2Type ofdrives,belt, gear and chain drive. 3.3 Computation of velocity ratio, length of belts (open and cross)with and without slip. 3.4 Ratio of belt tensions, centrifugal tension and initial tension.3.5Powertransmittedbythebelt.3.6Determinebeltthicknessandwidth forgivenpermissiblestressforopenandcrossedbeltconsideringcentrifugaltension.
3.7V-belts and V-belts pulleys.3.8 Conceptof crowningof pulleys.3.9 Gear drives anditsterminology.3.10Geartrains,workingprincipleofsimple,compound, reverted and epicyclic gear trains.

4.0GovernorsandFlywheel4.1Functionofgovernor4.2Classificationofgovernor
4.3WorkingofWatt,Porter,ProelandHartnellgovernors.4.4Conceptual explanation of sensitivity, stability and isochronisms. 4.5 Function of flywheel. 4.6 Comparisonbetweenflywheel&governor.4.7Fluctuationofenergyandcoefficient of fluctuation of speed.

5.0 Balancing of Machine 5.1 Concept of static and dynamic balancing. 5.2 Static balancingofrotatingparts.5.3Principlesofbalancingofreciprocatingparts.5.4 Causesandeffectofunbalance.5.5Differencebetweenstaticanddynamicbalancing

6.0 Vibration of machine parts 6.1 Introduction to Vibration and related terms(Amplitude, time period and frequency, cycle) 6.2 Classification of vibration. 6.3 Basicconceptofnatural,forced&dampedvibration6.4Torsionaland Longitudinalvibration.6.5Causes&remediesof vibration

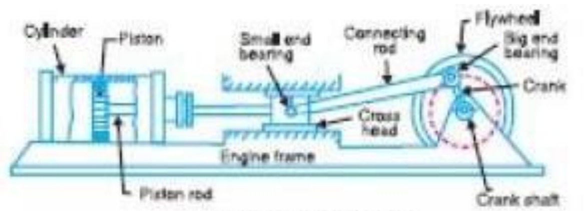
Module-1

WhatisKinematics?Kinematicsisthestudyofmotion(position,velocity,acceleration). A major goal of understanding kinematics is to develop the ability to design a system that will satisfy specified motion requirements. This will be the emphasis of this class.

WhatisKinetics? Kineticsisthestudyofeffectofforcesonmovingbodies.Good kinematic design should produce good kinetics.

DefinitionsLink:

A link is defined as a member or a combination of members of a mechanism connecting other members and having relative motion between them. The link may consist of one or more resistant bodies. A link may be called as kinematic link or element. Eg: Reciprocating steam engine.



Reciprocating steam engine.

Classificationoflinkisbinary,ternaryandquarternary.

Joint: Aconnectionbetweentwolinksthatallowsmotionbetweenthelinks.Themotion allowed may be rotational (revolute joint), translational (sliding or prismatic joint), or a combination of the two (roll-slide joint).

Kinematicpair: Kinematicpairisajointoftwolinkshavingrelativemotionbetween them. The types of kinematic pair are classified according to

- Nature of contact (lower pair, Higher pair)
- Nature of mechanical contact (Closed pair, unclosed pair)
- Nature of relative motion (Sliding pair, turning pair, rolling pair, screw pair, spherical pair)

classificationofkinematicspairs

<u>Accordingtothetypeofrelativemotionbetweentheelements</u>

a) Slidingpair:-Whentheelementsof apairareconnected in suchawaythatonecan only slide relative to the other, the pair is known as a slidingpair.

- Thepistonandcylinder,cross-head
- Guidesofareciprocatingsteamengine
- Ram and itsguidesinshaper
- Tailstockonthelathebed

(b) Turningpair:-Whentheelementsof apairareconnectedinsuchawaythat onecan onlyturn or revolve relative to another link, the pair isknown asturningpair.

- Ashaftwithcollarsatbothendsfittedintoacircularhole
- Thecrankshaftinajournal bearing inanengine
- Lathespindlesupportedinheadstock
- Cyclewheelsturningovertheiraxles

aretheexamplesofaturningpair

(c) Rolling pair:- When the elements of a pair are connected in such a way that one link rolls over another fixed link, the pair is known as rollingpair.

• Ballandrollerbearings

(d) Screw pair:- When the elements of a pair are connected in suchca way that one element turn about the other by screw threads, the pair isknown as screw pair.

• Theleadscrewofalathewithnut

• Boltwithanut

(e) Spherical pair:- When the elements of a pair are connected in such a way that one element turns or swivels about the other fixed element, the pair formed is called a spherical pair.

• Theballandsocketjoint

• Attachmentofacarmirror

• Penstand

Accordingtothetypeofcontactbetweenthe elements

(a) **Lower pair:-**When the elements of a pair having a surface contact between them when relativemotiontakes placeandthe surface of one element slides over the surface of the other, the pair formed is known as lowerpair.

• slidingpairs

• turningpairs

• screwpairs

(b) **Higher pair:-** When the elements of a pair having a line or point contact between them when relative motion takes place and the motion between the two elements is partly turning and partly sliding, then the pair isknown as higher pair.

• Pairoffrictiondiscs

• toothedgearing

• Beltandrope drives

Accordingtonatureofmechanicalconstraint

(a) **Closedpair:-**Whentheelementsofapairareheldtogethermechanicallyinsucha waythat only required relative motion occurs, it isthen knownasclosed pair.

• Thelowerpairsareclosedpair.

(b) **Unclosedpair/OpenPair:-**Whentheelementsofapairare in contacteitherdueto force of gravity or spring action, the pair is called as a Un-closed pair or OpenPair.

• The camand followerandgravity

Kinematic chain: When the kinematic pairsare coupled in such a way that the last link is joined to the first link to transmit definite motion it is called a kinematic chain. Eg: The crankshaftofanengineformsakinematicpairwiththebearingswhicharefixedinapair, the connecting rod with the crank forms a second kinematic pair, the piston with the connecting rod forms a third pair and the piston with the cylinder forms the fourth pair. The total combination of these links is a kinematic chain. Eg: Lawn mover

Here,we hadtocheckwhetherthegiven linkis akinematicchainWecanusetwo formulas

1. l= 2p-4

2. j=(3/2)l – 2

Mechanism Ifmotionofanyofthe movablelinksresultsindefinite motionsoftheothers the linkage is known as mechanism

MachineWhenamechanismis requiredtotransmitpowerortodo someparticulartype of work it then becomes a machine.

Degrees of Freedom It is defined as the number of input parameters which must be independently controlled in order to bring the mechanism in to useful engineering purposes. It is also defined as the number of independent relative motions, both translationalandrotational,apaircanhave.Degreesoffreedom=6 –no.ofrestraints.To find the number of degrees of freedom for a plane mechanism we have **Grubler's equation $F = 3(n - 1) - 2j_1 - j_2$**

F=Mobilityornumberofdegreesoffreedom n = Number of links including frame.

j_1=Jointswithsingle(one)degreeoffreedom. J_2 = Joints with two degrees of freedom.

F>0,resultsinamechanismwith'F'degreesoffreedom. F = 0, results in a statically determinate structure.

F<0,resultsinastaticallyindeterminatestructure

Grashoff'slaw:

Grashoff 4-bar linkage: A linkage that contains one or more links capable of undergoing a full rotation. A linkage is Grashoff if: S + L < P + Q (Where: S = shortest linklength,L=longest,P,Q=intermediatelengthlinks).Bothjointsoftheshortestlink are capable of 360 degrees of rotation in a Grashoff linkages. This gives us 4 possible linkages:

crank-rocker(inputrotates360

rocker-crank-rocker(couplerrotates360)

rocker-crank(follower)

doublecrank(alllinksrotate360).

InversionofMechanism

we can obtain as many mechanisms as the number of links in a kinematic chain by fixing, in turn, different links in a kinematic chain. This method of obtaining different mechanismsbyfixingdifferentlinksinakinematicchain,isknown as***inversionofthe mechanism***

TypesofKinematicChains

Themost important kinematic chains are those which consist of four lower pairs, each pair beingaslidingpairoraturningpair.Thefollowingthreetypesofkinematicchainswithfour lower pairs are important from the subject point of view :

1. Fourbarchainorquadriccyclicchain,

2. Singleslidercrankchain,and

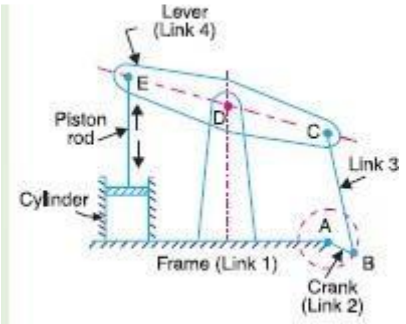
3. Doubleslidercrankchain.

InversionsofFourBarChain

Thoughtherearemanyinversionsofthefourbarchain,yetthefollowingareimportantfrom the subject point of view :

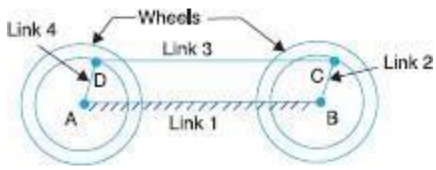
1. Beamengine(crankandlevermechanism).

Apartofthemechanismofabeamengine(alsoknownascrankandlevermechanism)which consists offourlinks, is shownin Fig. In this mechanism, whenthe crank rotatesaboutthe fixedcentre *A*, the leveroscillates about a fixedcentre *D*. Theend *E* ofthe lever*CDE* is connected to a piston rodwhich reciprocatesdue to the rotationofthe crank. In otherwords, the purpose of this mechanism is to convert rotary motion into reciprocating motion.

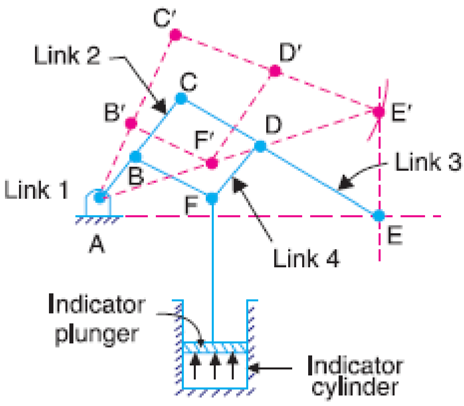


2. *Couplingrodofalocomotive*(*Doublecrankmechanism*).Themechanismofacoupling rodofa locomotive (also knownas double crank mechanism) which consists offour links, is shown in Fig.

Inthismechanism,thelinks*AD*and*BC*(havingequallength)actascranksandareconnected to the respective wheels.Thelink *CD* acts asa coupling rodandthe link *AB* is fixed in order tomaintainaconstantcentretocentredistancebetweenthem.Thismechanismismeantfor transmitting rotary motion from one wheel to the other wheel



3. **Watt'sstraight line mechanism or Double lever mechanism:** Inthis mechanism,the linksAB&DE act asleversattheendsA&E oftheseleversare fixed.TheAB&DE are parallel in the mean position of the mechanism and coupling rod BD is perpendicular to the levers AB & DE. On any small displacement of the mechanism the tracing point 'C' traces the shape of number '8', a portion of which will be approximately straight. Hence this is also an example for the approximate straight line mechanism. This mechanism is shown below



InversionsofSlidercrankChain: Itisafourbarchainhavingoneslidingpairandthree turning pairs. It is shown in the figure below the purpose of this mechanism is to convert rotary motion to reciprocating motion and vice versa.

Therearefourinversionsinasingleliderchainmechanism. They are: 1) Reciprocating engine mechanism (1st inversion)

2) Oscillatingcylinderenginemechanism(2ndinversion)

3) Crankandslottedlevermechanism(2ndinversion)

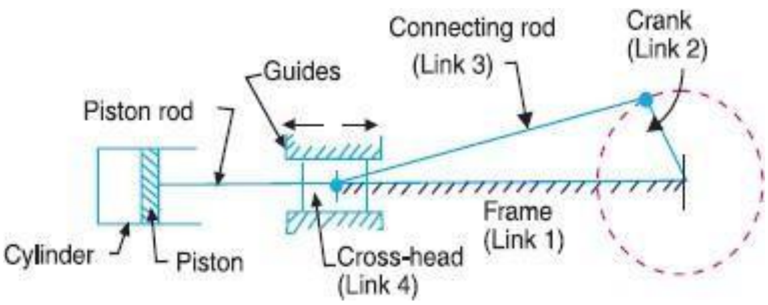
4) Whitworthquickreturnmotionmechanism(3rdinversion)

5) Rotaryenginemechanism(3rdinversion)

6) Bullenginemechanism(4thinversion)

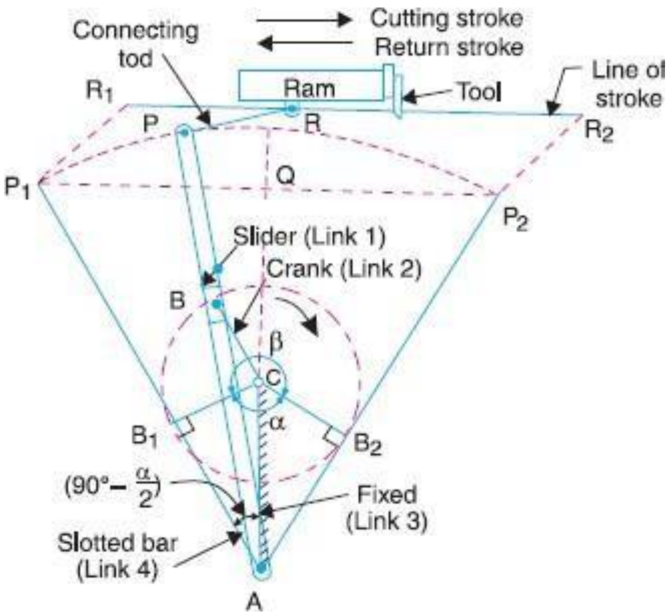
7) HandPump(4thinversion)

Reciprocatingenginemechanism: Inthefirstinversion,thelink1i.e.,thecylinderand the frame is kept fixed. The fig below shows a reciprocating engine



A slotted link 1 is fixed. When the crank 2 rotates about O, the sliding piston 4 reciprocatesintheslottedlink1.Thismechanismisusedinsteamengine,pumps, compressors, I.C. engines, etc.

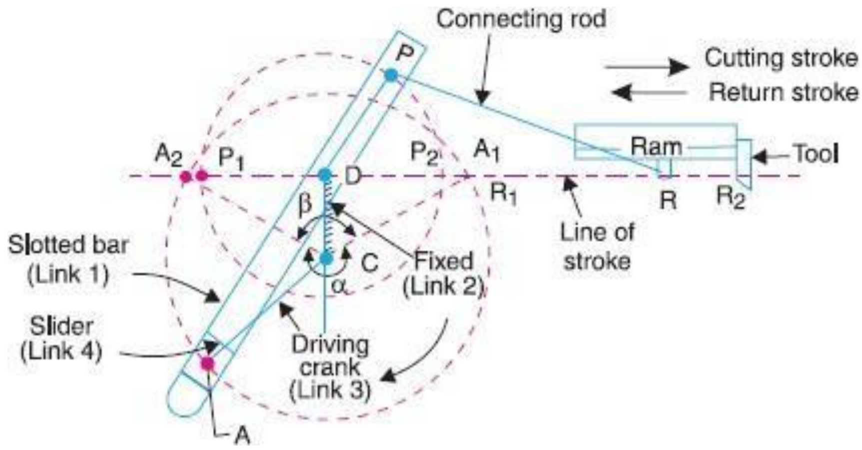
Crankandslottedlevermechanism: Itisanapplicationofsecondinversion.Thecrank and slotted lever mechanism is shown in figure below



In this mechanism link 3 is fixed. The slider (link 1) reciprocates in oscillating slotted lever (link 4) and crank (link 2) rotates. Link 5 connects link 4 to the ram (link 6). The ram with the cutting tool reciprocates perpendicular to the fixed link 3. The ram with the tool reverses its direction of motion when link 2 is perpendicular to link 4. Thus the cutting stroke is executed during the rotation of the crank through angle α and the return strokeisexecutedwhenthecrankrotatesthroughangleβor360 –α.Therefore,whenthe crank rotates uniformly, we get,

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360^\circ - \beta} \quad \text{or} \quad \frac{360^\circ - \alpha}{\alpha}$$

Whitworthquickreturnmotionmechanism

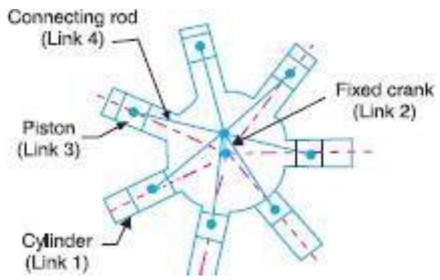


Third inversion is obtained by fixing the crank i.e. link 2. Whitworth quick return mechanism is an application of third inversion. This mechanism is shown in the figure below.ThecrankOCisfixedandOQrotatesaboutO.Thesliderslidesintheslottedlink and generates a circle of radius CP. Link 5 connects the extension OQ provided on the oppositesideofthelink1totheram(link6).TherotarymotionofPistakentotheramR which reciprocates. The quick return motion mechanism is used in shapers and slotting machines. The angle covered during cutting stroke from P1 to P2 in counter clockwise direction is α or 360 -2θ. During the return stroke, the angle covered is 2θ or β.

$$\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360^\circ - \beta} \quad \text{or} \quad \frac{360^\circ - \alpha}{\alpha}$$

RotaryenginemechanismorGnome Engine:

Rotary engine mechanism or gnome engine is another application of third inversion. It isa rotary cylinder V – type internal combustion engine used as an aero – engine. But now GnomeenginehasbeenreplacedbyGasturbines.TheGnomeenginehasgenerallyseven cylindersinoneplane.ThecrankOAisfixedandalltheconnectingrodsfromthepistons are connected to A. In this mechanism when the pistons reciprocate in the cylinders, the whole assembly of cylinders, pistons and connecting rods rotate about the axis O, where the entire mechanical power developed, is obtained in the form of rotation of the crank shaft. This mechanism is shown in the figure below

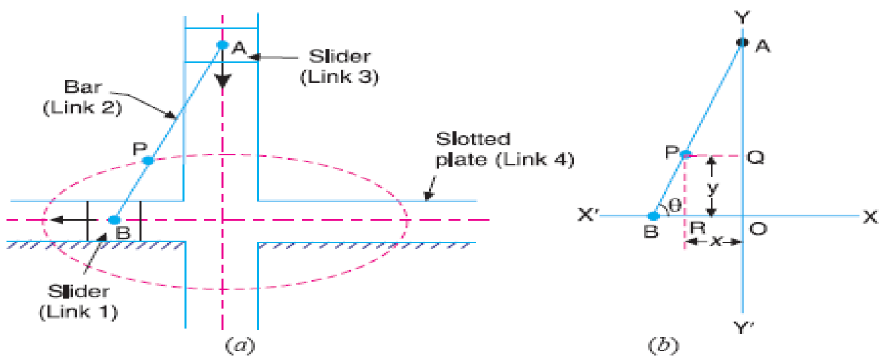


DoubleSliderCrankChain: Afourbarchainhavingtwoturningandtwoslidingpairs such that two pairs of the same kind are adjacent isknown as double slider crank chain.

Inversions of Double slider Crank chain: It consists of two sliding pairs and two turning pairs. There are three important inversions of double slider crank chain.

1) Ellipticaltrammel.
2) Scotchyokemechanism.
3) Oldham’sCoupling.

EllipticalTrammel: Thisisaninstrumentfordrawingellipses.Heretheslottedlinkis fixed.The sliding block Pand Q in vertical and horizontal slots respectively.The end R generates an ellipse with the displacement of sliders P and Q.



Let us take OX and OY as horizontal and vertical axes and let the link BA is inclined at an angle θ with the horizontal, as shown in Fig. 5.34 (*b*). Now the co-ordinates of the point P on the link BA will be

$$x = PQ = AP\cos\theta;\ \text{and}\ y = PR = BP\sin\theta$$

or
$$\frac{x}{AP} = \cos\theta;\ \text{and}\ \frac{y}{BP} = \sin\theta$$

Squaring and adding,

$$\frac{x^2}{(AP)^2} + \frac{y^2}{(BP)^2} = \cos^2\theta + \sin^2\theta = 1$$

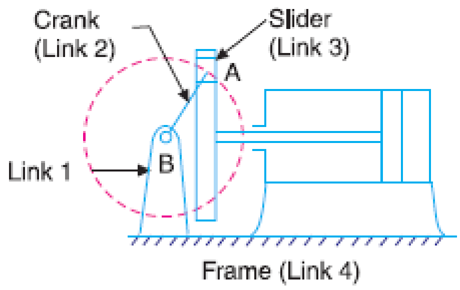
This is the equation of an ellipse. Hence the path traced by point P is an ellipse whose semi-major axis is AP and semi-minor axis is BP.

Note : If P is the mid-point of link BA, then $AP = BP$. The above equation can be written as

$$\frac{x^2}{(AP)^2} + \frac{y^2}{(AP)^2} = 1 \qquad \text{or} \qquad x^2 + y^2 = (AP)^2$$

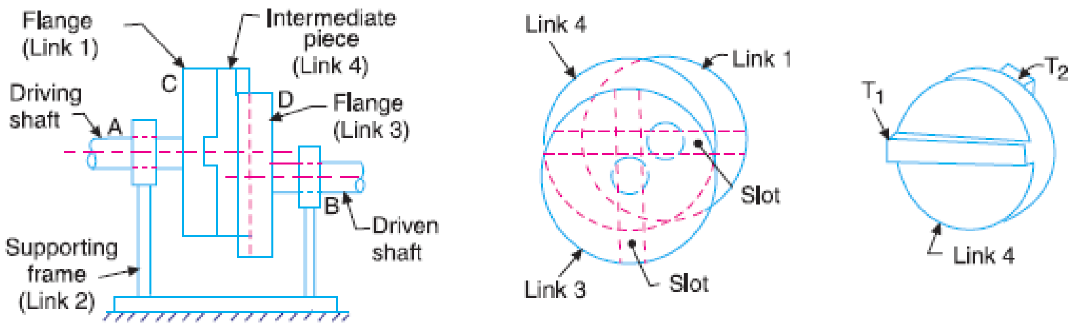
This is the equation of a circle whose radius is AP. Hence if P is the mid-point of link BA, it will trace a circle.

Scotch yoke mechanism: This mechanism is used to convert rotary motion in to reciprocatingmotion.Theinversionisobtainedbyfixingeitherthelink1orlink3.LinkI is fixed. In this mechanism when the link 2 rotates about B as centre, the link 4 reciprocates. The fixed link 1 guides the frame.



Scotch yoke mechanism.

Oldham's coupling: The third inversion of obtained by fixing the link connecting the 2 blocksP&Q.Ifoneblockisturningthroughanangle,theframeandtheotherblockwill also turn through the same angle. It is shown in the figure below



An application of the third inversion of the double slider crank mechanism is Oldham's coupling shown in the figure. This coupling is used for connecting two parallel shafts when the distance between the shafts is small. The two shafts to be connected have flangesattheirends, securedbyforging.Slotsarecutintheflanges.Theseflangesform1 and3. An intermediate disc having tongues at right angles and opposite sides is fitted in between the flanges. The intermediate piece forms the link4 which slides or reciprocates in flanges 1 & 3. The link two is fixed as shown.When flange 1 turns, the intermediate disc 4 must turn through the same angle and whatever angle 4 turns, the flange 3 must turnthroughthesameangle.Hence1,4&3musthavethesameangularvelocityatevery instant. If the distance between the axis of the shaft is x,it will be thediameter if the circle traced by the centre of the intermediate piece. The maximum sliding speed of each tongue along its slot is givenby

$$v=x\omega$$

where,

ω=angularvelocityofeachshaftinrad/sec v = linear velocity in m/sec

Camandfollowers

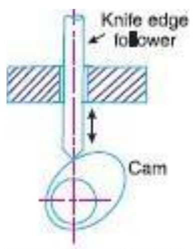
A ***cam*** is a rotating machine element which gives reciprocating or oscillating motiontoanother element known as ***follower*****.**Thecamandthefollowerhave a line contact and constitute a higher pair. The cams are usually rotated at uniformspeedbya shaft,but the followermotionispredeterminedandwillbe according to the shape of thecam.

ClassificationofFollowers

Thefollowersmaybeclassifiedasdiscussedbelow :

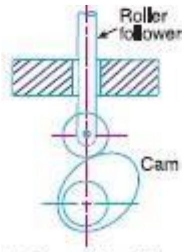
1. ***According to the surface in contact*****.**The followers, according to the surface in contact, are as follows :

(a) ***Knife edge follower*****.** When the contacting end ofthe follower has asharp knife edge, it is called a knife edge follower, The sliding motion takes place betweenthecontactingsurfaces(*i.e.*theknifeedgeandthecamsurface).It is seldomusedinpracticebecausethesmallareaofcontactingsurfaceresultsin excessive wear. In knife edge followers, a considerable side thrust exists between the follower and the guide.



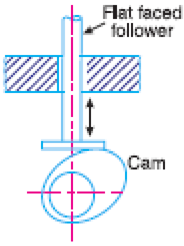
(*a*) Cam with knife edge follower.

(b) ***Roller follower*****.** When the contacting end of the follower is a roller, it is called a roller follower, Since the rolling motion takes place between the contacting surfaces (*i.e.* the roller and the cam), therefore the rate of wear is greatly reduced. In roller followers also the side thrust exists between the follower and the guide. The roller followers are extensively used where more space is available such as in stationary gas and oil engines and aircraft engines.



(*b*) Cam with roller follower.

(c) ***Flat faced or mushroom follower*****.**When the contacting end of the follower is a perfectly flat face, it is called a flat-faced follower.The flat faced followers are generally used where space is limited such as in cams which operate the valves of automobileengines.



(*c*) Cam with flat faced follower.

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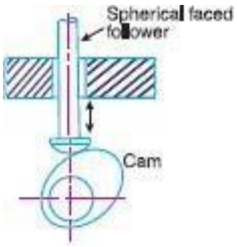
(d) ***Spherical faced follower*****.** When the contacting end of the follower is of spherical shape, it is called a spherical faced follower, It may be noted that whenaflat-facedfollowerisusedinautomobileengines,highsurfacestresses areproduced.Inordertominimisethesestresses,theflatendofthefolloweris machined to a spherical shape.



(*d*) Cam with spherical faced follower.

According to the motion of the follower**.**The followers, according to its motion, are of the following two types:

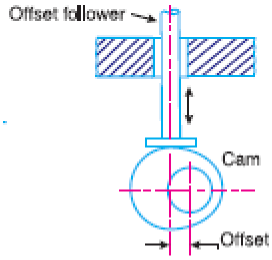
(a) ***Reciprocating ortranslating follower*****.** Whenthefollowerreciprocatesin guidesasthe cam rotatesuniformly,itisknownasreciprocatingor translating follower.

(b) ***Oscillating or rotating follower*****.** When the uniform rotary motion of the cam is converted into predetermined oscillatory motion of the follower, it is called oscillating or rotating follower.

3. ***Accordingtothepathofmotionofthefollower*****.**Thefollowers,according to its path of motion, are of the following two types:

(a) ***Radial follower*****.** When the motion ofthe follower is along an axis passing through the centre of the cam, it is known as radialfollower

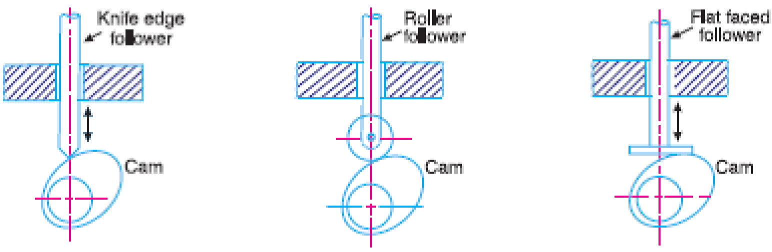
(b) ***Off-set follower*****.** When the motion of the follower is along an axis away from the axis of the cam centre, it is called off-setfollower.



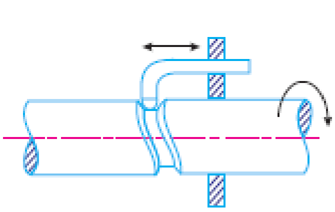
ClassificationofCams

Thoughthecamsmaybeclassifiedinmanyways,yetthefollowingtwotypesareimportant from the subject point of view

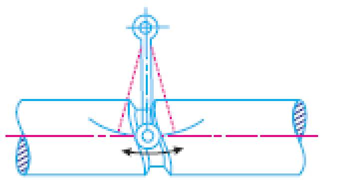
1. ***Radialordisccam*****.**Inradialcams,thefollowerreciprocatesoroscillatesinadirection perpendicular to the cam axis.Thecams as shownin Fig areallradialcams.



2. ***Cylindrical cam*****.** Incylindricalcams, the followerreciprocates oroscillates in a direction paralleltothecam axis.Thefollowerrides inagroove atitscylindricalsurface.Acylindrical groovedcam witha reciprocating and anoscillating followeris showninFig



(*a*) Cylindrical cam with reciprocating follower.



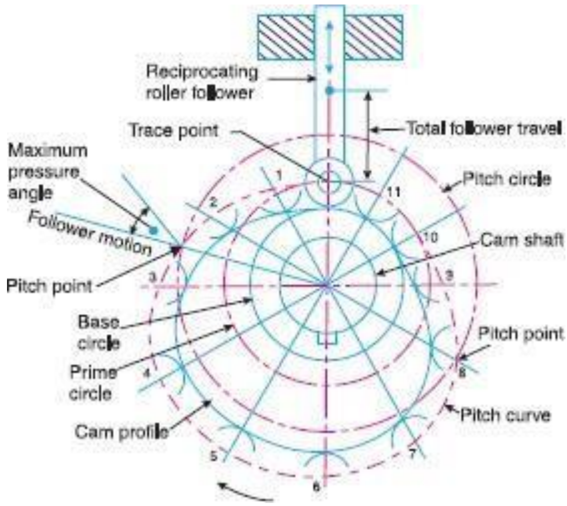
(*b*) Cylindrical cam with oscillating follower.

TermsUsedinRadial Cams

1. ***Basecircle*****.**Itisthesmallestcirclethatcanbedrawntothecamprofile.

2. ***Tracepoint*****.**Itisareferencepointonthefollowerandisusedtogeneratethe*pitchcurve*. In case ofknife edge follower, the knife edge represents the trace point andthe pitchcurve

correspondstothecamprofile.Inaroller follower,thecentreoftherollerrepresentsthe trace point.
3. ***Pressure angle*****.** It is the angle betweenthe direction ofthe follower motion and a normal tothepitchcurve.Thisangleisveryimportantindesigningacamprofile.Ifthepressureangle is too large, a reciprocating follower willjam in itsbearings.



4. ***Pitchpoint*****.**Itisapointonthepitchcurvehavingthemaximumpressureangle.
5. ***Pitchcircle*****.**Itisacircledrawnfromthecentreofthecamthroughthepitchpoints.
6. ***Pitch curve*****.** Itis the curve generated by the trace point as the followermoves relative to thecam.Foraknifeedgefollower,thepitchcurveandthecamprofilearesamewhereasfora roller follower, they are separated by the radius of theroller.
7. ***Prime circle*****.** Itis the smallestcircle that can be drawnfromthe centre ofthe cam and tangent to the pitch curve.For a knife edge and a flatfacefollower, the prime circle and the basecircleareidentical.Forarollerfollower,theprimecircleis largerthanthebasecircleby the radius of the roller.
8. ***Liftorstroke*****.**Itisthemaximumtravelofthefollowerfromitslowestpositiontothe Top most position.

Module2

2.Friction2.1Frictionbetweennutandscrewforsquarethread
screw jack 2.2 Bearing and its classification Description of roller needle roller& ball bearings.2.3Torquetransmissioninflatpivot&conicalpivotbearings.2.4Flat collar bearing of single and multiple types. 2.5 Torque transmission for single and multiple clutches 2.6 Working of simple frictional brakes.2.7 Working of Absorption type of dynamometer

NORMALFORCES

Whenanobjectrestsonasurface,thesurfaceexertsanormalforceontheobject,keepingit from accelerating downward.

A normal force is perpendicular to the contact surface of an object. Example – When we are standing, we do not fall through the floor because the downward force of our weight is balanced bytheupwardnormalforceexertedbythesurfaceofthefloor.Themagnitudesoftheseforcesare equal, but they are applied in opposite directions. However, if we stood on a piece of paper, the normalforce ofthe paperwould notbe greatenough to counteractourweightbecausethe paperis not strong enough. The forces would be unbalanced and we would accelerate downward, falling through the paper

FRICTION

It is harder to move objects with larger inertia, but there are ways to make moving objects with larger amounts of inertia easier. One way is to reduce the amount of friction between the object and its contact surface.

Friction is a force that resists the relative motion of two objects in contact, caused by the irregularities of the surfaces coming into contact and colliding with each other.

Therearetwotypesoffrictiontoconsider:

- **Static friction** is the force that opposes the start of relative motion between two objects in contact with each other.
- **Kinetic friction** is the force that opposes the relative motion between two objects in contactwith each other when the objects are actually in motion.

Importantthingstoknowaboutfriction:

1. Frictionisalwaysparalleltothecontactsurfaceand is intheoppositedirectionoftheforce causing the motion

2. Staticfrictionisalwaysgreaterthankineticfriction.

- Thisisduetoinertia– Anobject atresttendstostayatrest while anobjectin motiontendsto continue moving.

3. Frictionincreasesastheforcebetweentwosurfaces increases.

- Frictiondependsonthenatureofthematerialscomingintocontactwitheachother.
- Frictiondependsontheforcepressingtheobjectstogether

LawsofStaticFriction

Followingarethelawsofstaticfriction:

1. Theforceoffrictionalwaysactsinadirection,oppositetothatinwhichthebodytendsto move.

2. Themagnitudeoftheforceoffrictionisexactlyequaltotheforce,whichtendsthebody to move.

3. Themagnitudeofthelimitingfriction(F)bearsaconstantratiotothenormalreaction (R_N) between the two surfaces. Mathematically

$$F/R_N=\text{constant}$$

4. Theforceoffrictionisindependentoftheareaofcontact,betweenthetwosurfaces.

5. Theforceoffrictiondependsupontheroughnessofthesurfaces

LawsofKineticorDynamicFriction

1. Theforceoffrictionalwaysactsinadirection,oppositetothatinwhichthebodyis moving.

2. Themagnitudeofthekineticfrictionbearsaconstantratiotothenormalreactionbetween the twosurfaces. But this ratio is slightly less than that in case oflimiting friction.

3. Formoderatespeeds,theforceoffrictionremainsconstant.Butitdecreasesslightlywith the increase of speed.

LawsofSolidFriction

1. Theforceoffrictionisdirectlyproportionaltothenormalloadbetweenthesurfaces.

2. Theforceoffrictionisindependentoftheareaofthecontactsurfaceforagivennormal load.

3. Theforceoffrictiondependsuponthematerialofwhichthecontactsurfacesaremade.

4. Theforceoffrictionisindependentofthevelocityofslidingofonebodyrelativetothe other body.

LawsofFluidFriction

1. Theforceoffrictionisalmostindependentofthe load.

2. Theforceoffrictionreduceswiththeincreaseofthetemperatureofthelubricant.

3. Theforceoffrictionisindependentofthesubstancesofthebearingsurfaces.

4. Theforceoffrictionisdifferentfordifferentlubricants.

CoefficientofFriction

Itisdefinedastheratioofthelimitingfriction(F)tothenormalreaction(R_N)betweenthetwo bodies. It is generally denoted by μ. Mathematically, coefficient of friction,

$$\mu = F/R_N$$

Angleof Repose

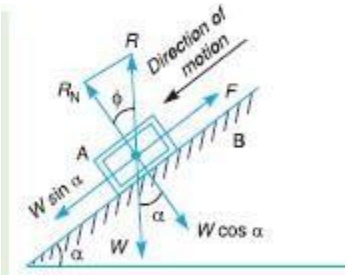
Considerthatabody *A*of weight(W) isrestingonaninclinedplane *B*,asshowninFig.10.3.If the angle of inclination $\alpha$of the plane to the horizontal is such that the body begins to move down the plane, then the angle φ is called the ***angle of repose***.

Alittleconsiderationwillshowthatthebodywillbegintomovedowntheplanewhentheangle of inclination of the plane is equal to the angle of friction (*i.e.* $\alpha = \varphi$). This may be proved as follows :

Theweightofthebody(W)canberesolvedintothefollowingtwocomponents:

1. Wsinα,paralleltotheplane*B*.Thiscomponenttendstoslidethebodydown the plane.

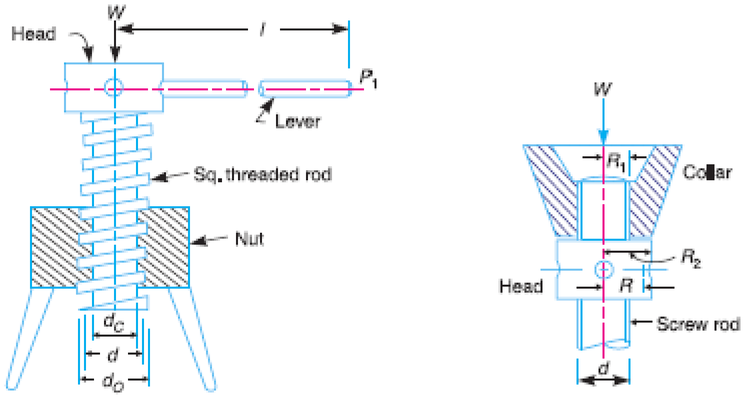
2. Wcosα,perpendiculartotheplane*B*.Thiscomponentisbalancedbythenormal reaction(R_N)ofthebody *A*andtheplane*B*.Thebodywillonlybegintomovedowntheplane, when



$$W \sin\alpha = F = \mu . R_N = \mu . W \cos\alpha$$
$$\tan\alpha = \mu = \tan\phi \quad \text{or} \quad \alpha = \phi$$

Screw Jack

Thescrewjackisadevice,forliftingheavyloads,byapplyingacomparativelysmallereffortat its handle. The principle, on which a screw jack works is similar to that of an inclined plane

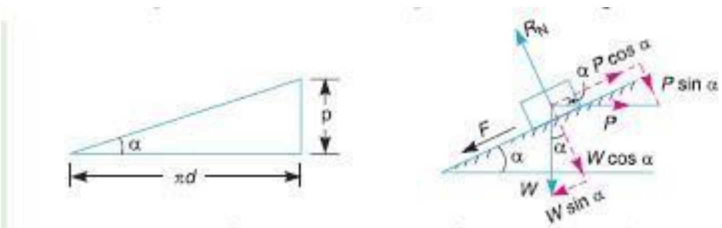


(a) Screw jack. (b) Thrust collar.

Fig(a)showsacommonformofascrewjack,whichconsistsofasquarethreaded rod (alsocalledscrew rodorsimplyscrew)whichfits intotheinnerthreads ofthenut. Theload,to beraisedorlowered, is placed onthe head ofthesquare threaded rodwhich is rotatedby the application ofaneffort atthe end ofthe lever for lifting orloweringthe load.

TorqueRequiredtoLifttheLoadbyaScrewJack

Ifonecompleteturn ofascrewthreadbyimaginedto beunwound, fromthe body ofthe screw and developed, it willform aninclined plane as shownin Fig



(a) Development of a screw. (b) Forces acting on the screw.

p=Pitchofthescrew,

d=Meandiameterofthescrew,

α=Helixangle,

P=Effortappliedatthecircumferenceofthescrewtoliftthe load,

W=Loadtobelifted,and

μ=Coefficientoffriction,betweenthescrewandnut=tanφ, where

φ is the friction angle

$\tan \alpha = p/\pi d$

Resolving the forces along the plane,

$$P \cos \alpha = W \sin \alpha + F = W \sin \alpha + \mu . R_N$$

and resolving the forces perpendicular to the plane,

$$R_N = P \sin \alpha + W \cos \alpha$$

Substituting this value of R_N in equation (*i*),

$$P \cos \alpha = W \sin \alpha + \mu (P \sin \alpha + W \cos \alpha)$$

$$= W \sin \alpha + \mu P \sin \alpha + \mu W \cos \alpha$$

or $$P \cos \alpha - \mu P \sin \alpha = W \sin \alpha + \mu W \cos \alpha$$

or $$P (\cos \alpha - \mu \sin \alpha) = W (\sin \alpha + \mu \cos \alpha)$$

$$\therefore \quad P = W \times \frac{\sin\alpha + \mu\cos\alpha}{\cos\alpha - \mu\sin\alpha}$$

Substituting the value of $\mu = \tan\phi$ in the above equation, we get

$$P = W \times \frac{\sin\alpha + \tan\phi\cos\alpha}{\cos\alpha - \tan\phi\sin\alpha}$$

Multiplying the numerator and denominator by $\cos\phi$,

$$P = W \times \frac{\sin\alpha\cos\phi + \sin\phi\cos\alpha}{\cos\alpha\cos\phi - \sin\alpha\sin\phi} = W \times \frac{\sin(\alpha+\phi)}{\cos(\alpha+\phi)}$$

$$= W\tan(\alpha+\phi)$$

$\therefore$ Torque required to overcome friction between the screw and nut,

$$T_1 = P \times \frac{d}{2} = W\tan(\alpha+\phi)\frac{d}{2}$$

theaxialloadistakenupbyathrustcollaroraflatsurface,asshowninFig. sothattheloaddoesnotrotatewiththescrew,thenthetorquerequiredtoovercomefrictionat the collar

$$T_2 = \mu_1 . W\left(\frac{R_1 + R_2}{2}\right) = \mu_1 . W . R$$

R_1 and R_2 = Outside and inside radii of the collar,

R = Mean radius of the collar, and

μ_1 = Coefficient of friction for the collar.

. Total torque required to overcome friction (*i.e.* to rotate the screw),

$$T = T_1 + T_2 = P \times \frac{d}{2} + \mu_1 . W . R$$

If an effort P_1 is applied at the end of a lever of arm length l, then the total torque required to vercome friction must be equal to the torque applied at the end of the lever, *i.e.*

$$T = P \times \frac{d}{2} = P_1 . l$$

Example-1

A150mmdiametervalve,againstwhichasteampressureof2MN/m2is acting,isclosedby means ofasquarethreadedscrew50mminexternaldiameterwith6mm pitch.If the coefficient of friction is 0.12 ; find the torque required to turn the handle.

Solution. Given : D = 150 mm = 0.15 mm = 0.15 m ; Ps = 2 MN/m^2 = 2 × 10^6 N/m^2 ; d_0 = 50 mm ; p = 6 mm ; $\mu = \tan\phi = 0.12$

We know that load on the valve,

$$W = \text{Pressure} \times \text{Area} = p_S \times \frac{\pi}{4} D^2 = 2 \times 10^6 \times \frac{\pi}{4}(0.15)^2 \text{ N}$$

$$= 35\ 400 \text{ N}$$

Mean diameter of the screw,

$$d = d_0 - p/2 = 50 - 6/2 = 47 \text{ mm} = 0.047 \text{ m}$$

$$\therefore \quad \tan\alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 47} = 0.0406$$

We know that force required to turn the handle,

$$P = W \tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha . \tan\phi}\right]$$

$$= 35400\left[\frac{0.0406 + 12}{1 - 0.0406 \times 0.12}\right] = 5713 \text{ N}$$

∴ Torque required to turn the handle,

$$T = P \times d/2 = 5713 \times 0.047/2 = 134.2 \text{ N-m} \quad \textbf{Ans.}$$

Example-2

Thecutterofabroachingmachineispulledbysquarethreadedscrewof 55 mmexternaldiameterand10mmpitch.Theoperatingnuttakestheaxialloadof400Nona flatsurface of 60 mm internal diameter and 90 mm external diameter. If the coefficient of firction is 0.15 for all contact surfaces on the nut, determine the power required to rotate the operating nut, when the cutting speed is 6 m/min.

Solution. Given : d_0 = 55 mm ; p = 10 mm = 0.01 m ; W = 400 N ; D_2 = 60 mm or R_2 = 30 mm ; D_1 = 90 mm or R_1 = 45 mm ; $\mu = \tan\phi = \mu_1 = 0.15$

We know that mean diameter of the screw,

$$d = d_0 - p/2 = 55 - 10/2 = 50 \text{ mm}$$

$$\therefore \quad \tan\alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 50} = 0.0637$$

and force required at the circumference of the screw,

$$P = W \tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha . \tan\phi}\right]$$

$$= 400\left[\frac{0.0637 + 0.15}{1 - 0.0637 \times 0.15}\right] = 86.4 \text{ N}$$

We know that mean radius of the flat surface,

$$R = \frac{R_1 + R_2}{2} = \frac{45 + 30}{2} = 37.5 \text{ mm}$$

∴ Total torque required,

$$T = P \times \frac{d}{2} + \mu_1 . W . R = 86.4 \times \frac{50}{2} + 0.15 \times 400 \times 37.5 \text{ N-mm}$$

$$= 4410 \text{ N-mm} = 4.41 \text{ N-m} \qquad ...(\because \mu_1 = \mu)$$

Since the cutting speed is 6 m/min, therefore speed of the screw,

$$N = \frac{\text{Cutting speed}}{\text{Pitch}} = \frac{6}{0.01} = 600 \text{ r.p.m.}$$

and angular speed, $\omega = 2\pi \times 600/60 = 62.84$ rad/s

We know that power required to operate the nut

$$= T.\omega = 4.41 \times 62.84 = 277 \text{ W} = 0.277 \text{ kW} \quad \textbf{Ans.}$$

TorqueRequiredtoLowertheLoadbyaScrew Jack

Ifonecompleteturnofascrewthreadbeimaginedtobeunwoundfromthebody ofthescrew and developed, it will form an inclined plane as shown in Fig.

Let p=Pitchofthescrew,

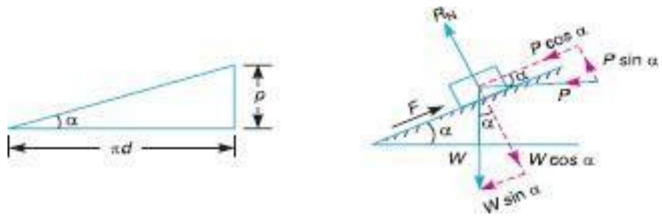
d=Meandiameterofthescrew,

α=Helixangle,

P=Effortappliedatthecircumferenceofthescrewtolowertheload

W=Loadtobelifted,and

μ=Coefficientoffriction,betweenthescrewandnut=tanφ, where
φ is the friction angle



From the geometry of the figure, we find that

$$\tan \alpha = p/\pi d$$

Resolving the forces along the plane,

$$P \cos \alpha = F - W \sin \alpha = \mu . R_N - W \sin \alpha$$

and resolving the forces perpendicular to the plane,

$$R_N = W \cos \alpha - P \sin \alpha$$

Substituting this value of R_N in equation *(i)*,

$$P \cos \alpha = \mu\ (W \cos \alpha - P \sin \alpha) - W \sin \alpha$$

$$= \mu . W \cos \alpha - \mu . P \sin \alpha - W \sin \alpha$$

or $$P \cos \alpha + \mu . P \sin \alpha = \mu . W \cos \alpha - W \sin \alpha$$

or $$P (\cos \alpha + \mu \sin \alpha) = W\ (\mu \cos \alpha - \sin \alpha)$$

$$\therefore \quad P = W \times \frac{(\mu \cos \alpha - \sin \alpha)}{(\cos \alpha + \mu \sin \alpha)}$$

Substituting the value of $\mu = \tan \phi$ in the above equation, we get

$$P = W \times \frac{(\tan \phi \cos \alpha - \sin \alpha)}{(\cos \alpha + \tan \phi \sin \alpha)}$$

Multiplying the numerator and denominator by $\cos \phi$,

$$P = W \times \frac{(\sin \phi \cos \alpha - \sin \alpha \cos \phi)}{(\cos \alpha \cos \phi + \sin \phi \sin \alpha)} = W \times \frac{\sin (\phi - \alpha)}{\cos (\phi - \alpha)}$$

$$= W \tan (\phi - \alpha)$$

$\therefore$ Torque required to overcome friction between the screw and nut,

$$T = P \times \frac{d}{2} = W \tan (\phi - \alpha) \frac{d}{2}$$

Example-3

Themeandiameterofasquarethreadedscrewjackis50mm.Thepitchof thethreadis10mm.Thecoefficientoffrictionis0.15.Whatforcemustbeappliedattheendof a0.7mlonglever,whichisperpendiculartothelongitudinalaxisofthescrewtoraisealoadof 20 kN and to lower it?

Solution. Given : $d = 50$ mm $= 0.05$ m ; $p = 10$ mm ; $\mu = \tan\phi = 0.15$; $l = 0.7$ m ; $W = 20$ kN $= 20 \times 10^3$ N

We know that $\tan\alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 50} = 0.0637$

Let P_1 = Force required at the end of the lever.

Force required to raise the load

We know that force required at the circumference of the screw,

$$P = W\tan(\alpha + \phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha.\tan\phi}\right]$$

$$= 20 \times 10^3\left[\frac{0.0637 + 0.15}{1 - 0.0637 \times 0.15}\right] = 4314 \text{ N}$$

Now the force required at the end of the lever may be found out by the relation,

$$P_1 \times l = P \times d/2$$

$$\therefore \quad P_1 = \frac{P \times d}{2\,l} = \frac{4314 \times 0.05}{2 \times 0.7} = 154 \text{ N} \quad \textbf{Ans.}$$

Force required to lower the load

We know that the force required at the circumference of the screw,

$$P = W\tan(\phi - \alpha) = W\left[\frac{\tan\phi - \tan\alpha}{1 + \tan\phi.\tan\alpha}\right]$$

$$= 20 \times 10^3\left[\frac{0.15 - 0.0637}{1 + 0.15 \times 0.0637}\right] = 1710 \text{ N}$$

Now the force required at the end of the lever may be found out by the relation,

$$P_1 \times l = P \times \frac{d}{2} \quad \text{or} \quad P_1 = \frac{P \times d}{2l} = \frac{1710 \times 0.05}{2 \times 0.7} = 61 \text{ N} \quad \textbf{Ans.}$$

EfficiencyofaScrewJack

The efficiency of a screw jack may be defined as **the ratio between the ideal effort** (*i.e.* the effortrequiredtomovetheload,neglectingfriction)to **theactualeffort**(*i.e.*theeffortrequired to move the load taking friction into account).

Weknowthattheeffortrequiredtoliftttheload(W)whenfrictionistakenintoaccount

$$P = W\tan(\alpha + \phi) \quad \ldots($$

α = Helix angle,

ϕ = Angle of friction, and

, μ = Coefficient of friction, between the screw and nut $= \tan\phi$

Iftherewouldhavebeennofrictionbetweenthescrewandthenut,thenφwillbeequalto zero. The value of effort P_0 necessary to raise the load, will then be given by the equation

$$P_0 = W\tan\alpha$$

$$\therefore \text{ Efficiency, } \eta = \frac{\text{Ideal effort}}{\text{Actual effort}} = \frac{P_0}{P} = \frac{W\tan\alpha}{W\tan(\alpha + \phi)} = \frac{\tan\alpha}{\tan(\alpha + \phi)}$$

MaximumEfficiencyofaScrew Jack

$$\eta = \frac{\tan\alpha}{\tan(\alpha + \theta)} = \frac{\frac{\sin\alpha}{\cos\alpha}}{\frac{\sin(\alpha + \phi)}{\cos(\alpha + \phi)}} = \frac{\sin\alpha \times \cos(\alpha + \phi)}{\cos\alpha \times \sin(\alpha + \phi)}$$

$$= \frac{2\sin\alpha \times \cos(\alpha + \phi)}{2\cos\alpha \times \sin(\alpha + \phi)}$$

$$= \frac{\sin(2\alpha + \phi) - \sin\phi}{\sin(2\alpha + \phi) + \sin\phi}$$

$$\sin(2\alpha + \phi) = 1 \quad \text{or} \quad \text{when } 2\alpha + \phi = 90^\circ$$

$$2\alpha = 90^\circ - \phi \quad \text{or} \quad \alpha = 45^\circ - \phi/2$$

$$\eta_{max} = \frac{\sin(90^\circ - \phi + \phi) - \sin\phi}{\sin(90^\circ - \phi + \phi) + \sin\phi} = \frac{\sin 90^\circ - \sin\phi}{\sin 90^\circ + \sin\phi} = \frac{1 - \sin\phi}{1 + \sin\phi}$$

OverHaulingandSelfLocking Screws

$$P = W\tan(\phi - \alpha)$$

$$T = P \times \frac{d}{2} = W\tan(\phi - \alpha)\frac{d}{2}$$

Intheaboveexpression, if φ<α,thentorquerequired to lowerthe loadwillbe***negative.***In otherwords,theloadwillstartmoving downwardwithouttheapplicationofanytorque.Sucha conditionis knownas***over hauldingof screws.*** If however,φ >α,thetorquerequiredto lowerthe loadwill***positive***, indicating thataneffortis applied to lowerthe load. Such ascrew isknownas***selflockingscrew.***Inotherwords,ascrewwillbeselflockingifthefrictionangle isgreaterthanhelixangleorcoefficientoffrictionisgreaterthantangentofhelixangle*i.e.*μor tan φ > tan α

EfficiencyofSelfLockingScrews

$$\eta = \frac{\tan\alpha}{\tan(\alpha + \phi)}$$

and for self locking screws, $\phi \geq \alpha$ or $\alpha \leq \phi$.

∴ Efficiency of self locking screws,

$$\eta \leq \frac{\tan\phi}{\tan(\phi + \phi)} \leq \frac{\tan\phi}{\tan 2\phi} \leq \frac{\tan\phi(1 - \tan^2\phi)}{2\tan\phi}$$

$$\leq \frac{1}{2} - \frac{\tan^2\phi}{2} \qquad \ldots\left(\because \tan 2\phi = \frac{2\tan\phi}{1 - \tan^2\phi}\right)$$

From this expression we see that efficiency of self locking screws is less than $\frac{1}{2}$ or 50%. If the efficiency is more than 50%, then the screw is said to be overhauling.

BEARING

A**bearing**isamachineelementthatconstrainsrelativemotiontoonlythedesiredmotion, and reduces friction between moving parts

classification

Dependinguponthedirectionoftheforce

Radiabearing:Radialbearingsupportstheloadwhichisperpendiculartotheaxisoftheshaft

Thurstbearing: Thurstbearingsupportstheloadwhichactsalongtheaxisoftheshaft

Dependinguponthetypeoffriction

Slidingcontactbearing: Inthistypeofbearings,thesurfaceoftheshaftslidesoverthe surfaceofthebush.Topreventfriction,bothsurfacesareseparatedbyathinfilmoflubricating oil. Generally, Bushis made from bronze or whitemetal.

Example:Plainbearing,journalbearing,sleevebearing

Rollingcontactbearingoranti-frictionbearing:Hererollingfrictioncomes into play. This bearing is also called anti-friction bearing as friction is negligible which is in rangeof0.005to0.003fR.C.

Example:Bearingsusedinautomobileaxle,gearbox,machinetoolspindles,smallelectric motor

Types ofrollingcontactbearings

Deepgrooveballbearing:Inthistypeofbearings,theradiusoftheballisslightlyless thantheradiusof curvatureofthegroove.Thiscreatespointcontact.Thusfrictionislessand so itcanbeusedinhigh-speedapplications.Duetolowfriction,temperatureriseandthenoise levelisalsolow.Itcantakeaxialaswellasradialloads.

Cylindricalrollerbearing:Thesebearingareusedwherehighloadcarryingcapacityis required. Here rolling elements are cylindrical in shape instead of balls as in deep groove ball bearing.Itgiveslinecontactandsameaspreviouscasefrictionlossislesssoitcanbeusedin the high sped application. This bearing can not take thrust load.

Angularcontactbearing:Thistypeofbearingisdesignedinsuchawaythatline of reactionat thepointofcontactsforaball at theinnerraceandouterrace,makeananglewith the axisofthe bearing.Dueto this,it cantaketheradial andaxialloads simultaneously.Load carrying capacity of such bearings is high compared to deep groove ball bearings. However, two bearings are required to take thrust load in both directions.

Self-aligningbearing:Inthistypeofbearing,theexternalsurfaceofthebearingbushis madespherical.Thecentreofthissphericalsurfaceisatthecentreofthebearingsoitcanalign itself with the journal. Itis usedtocompensateanymisalignment.Itcantake both radialand axial loads.

Taperrollerbearing:Hererollingelementsarerollers.Theyarearrangedinsuchawaythat axesofindividual rolling elementsintersect at acommonpoint at theaxisofthebearing.Thisis forthepurerollingmotion.

Twobearingsarerequiredtotakeaxialloads.

Thistypeofbearingsiscommonlyusedinautomobile,railwayandmachine tools.

Thurstballbearing:Thurstballbearingconsisttworowofballs.Balls,innerand outer race are made from high carbon chromium steel while the roller is made from case hardened steel. These bearings are generally used in the gearbox.Due to a large number of balls, it can takecomparativelyhighthrustload.However,,itcantakethrustloadingsingledirectiononly

Frictioninjournalbearing

Let ϕ = Angle between R (resultant of F and R_N) and R_N,

μ = Coefficient of friction between the journal and bearing,

T = Frictional torque in N-m, and

r = Radius of the shaft in metres.

For uniform motion, the resultant force acting on the shaft must be zero and the resultant turning moment on the shaft must be zero. In other words,

$$R = W, \text{ and } T = W \times OC = W \times OB \sin\phi = W.r \sin\phi$$

Since ϕ is very small, therefore substituting $\sin\phi = \tan\phi$

$$\therefore \quad T = W.r \tan\phi = \mu.W.r \qquad \ldots(\because \mu = \tan\phi)$$

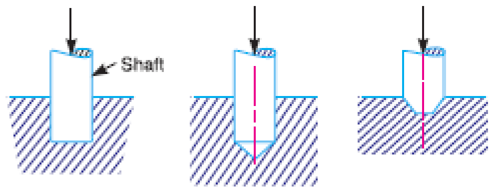
If the shaft rotates with angular velocity ω rad/s, then power wasted in friction,

$$P = T.\omega = T \times 2\pi N/60 \text{ watts}$$

where N = Speed of the shaft in r.p.m.

FrictionofPivotBearing

Thebearing surfacesplaced atthe end ofa shaft to take the axial thrust areknownas***pivots.***Thepivotmayhaveaflatsurfaceorconicalsurface.Whentheconeistruncated,itis then known as truncated or trapezoidal pivot.



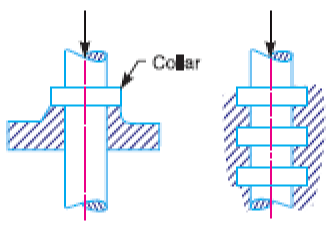
(*a*) Flat pivot. (*b*) Conical pivot. (*c*) Truncated pivot.

Consinderthetwoconditionforfriction

1. Thepressureisuniformlydistributedthroughoutthebearingsurface,and
2. Thewearisuniformthroughoutthebearingsurface

Frictionofcollarbearing

Thecollar may have flat bearing surface or conical bearing surface,but the flatsurface is mostcommonlyused.Theremaybeasinglecollar,asshowninFig.orseveralcollarsalong the length ofa shaft, as shownin Figin order to reduce the intensity of pressure.



(*d*) Single flat collar. (*e*) Multiple flat collar.

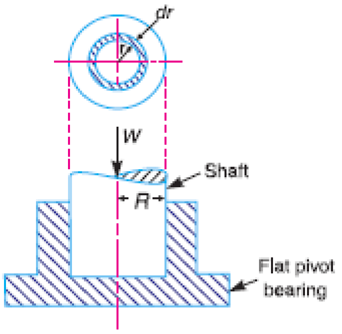
FlatPivot Bearing

Whenaverticalshaftrotatesinaflatpivotbearingasshownin Fig.,theslidingfrictionwillbe along the surface of contact between the shaft and the bearing.

Let*W*=Loadtransmittedoverthebearingsurface,

R=Radiusofbearingsurface,

p=Intensityofpressureperunitareaofbearingsurfacebetweenrubbingsurfaces,and μ=Coefficientoffriction.



Wewillconsiderthefollowingtwocases:

1. Whenthereisauniform pressure

When the pressure is uniformly distributed over the bearing area, then

$$p = \frac{W}{\pi R^2}$$

Consider a ring of radius r and thickness dr of the bearing area.

$\therefore$ Area of bearing surface, $A = 2\pi r.dr$

Load transmitted to the ring,

$$\delta W = p \times A = p \times 2\pi r.dr \qquad ...(i)$$

Frictional resistance to sliding on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W = \mu\, p \times 2\pi r.dr = 2\pi \mu.p.r.dr$$

$\therefore$ Frictional torque on the ring,

$$T_r = F_r \times r = 2\pi \mu\, p\, r.dr \times r = 2\pi \mu\, p\, r^2\, dr \qquad ...(ii)$$

Integrating this equation within the limits from 0 to R for the total frictional torque on the pivot bearing.

$\therefore$ Total frictional torque, $T = \int_0^R 2\pi\mu\, p r^2\, dr = 2\pi\mu\, p \int_0^R r^2\, dr$

$$= 2\pi\mu\, p\left[\frac{r^3}{3}\right]_0^R = 2\pi\mu\, p \times \frac{R^3}{3} = \frac{2}{3} \times \pi\mu.p.R^3$$

$$= \frac{2}{3} \times \pi\mu \times \frac{W}{\pi R^2} \times R^3 = \frac{2}{3} \times \mu.W.R \qquad ...\left(\because\ p = \frac{W}{\pi R^2}\right)$$

When the shaft rotates at ω rad/s, then power lost in friction,

$$P = T.\omega = T \times 2\pi N/60 \qquad ...(\because\ \omega = 2\pi N/60)$$

where N = Speed of shaft in r.p.m.

2. Whenthereisauniformwear

wear

$$p.r = C \text{ (a constant)} \quad \text{or} \quad p = C/r$$

and the load transmitted to the ring,

$$\delta W = p \times 2\pi r.dr \qquad \text{...[From equation } (i)]$$

$$= \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

$\therefore$ Total load transmitted to the bearing

$$W = \int_0^R 2\pi\, C.dr = 2\pi\, C[r]_0^R = 2\pi C.R \quad \text{or} \quad C = \frac{W}{2\pi R}$$

We know that frictional torque acting on the ring,

$$T_r = 2\pi\mu\, p\, r^2\, dr = 2\pi\mu \times \frac{C}{r} \times r^2\, dr \qquad \ldots\left(\because\ p = \frac{C}{r}\right)$$

$$= 2\pi\, \mu.C.r\ dr \qquad \ldots(iii)$$

$\therefore$ Total frictional torque on the bearing,

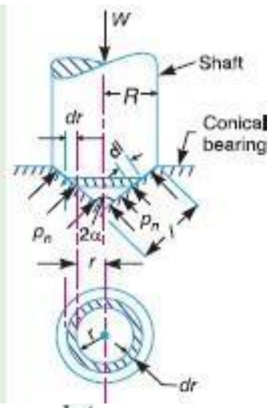
$$T = \int_0^R 2\pi\, \mu.C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_0^R$$

$$= 2\pi\mu.C \times \frac{R^2}{2} = \pi\mu.C.R^2$$

$$= \pi\mu \times \frac{W}{2\pi R} \times R^2 = \frac{1}{2} \times \mu.W.R \qquad \ldots\left(\because\ C = \frac{W}{2\pi R}\right)$$

ConicalPivotBearing

Theconicalpivotbearingsupportingashaftcarryingaload*W*isshowninFig



Let

P_n = Intensity of pressure normal to the cone,

α = Semi angle of the cone,

μ = Coefficient of friction between the shaft and the bearing, and

R = Radius of the shaft.

Consider a small ring of radius r and thickness dr. Let dl is the length of ring along the cone, such that

$$dl = dr \operatorname{cosec} \alpha$$

$\therefore$ Area of the ring,

$$A = 2\pi r.dl = 2\pi r.dr \operatorname{cosec} \alpha \qquad \ldots(\because\ dl = dr \operatorname{cosec} \alpha)$$

Consideruniformcondition

We know that normal load acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area}$$
$$= p_n \times 2\pi r.dr \text{ cosec } \alpha$$

and vertical load acting on the ring,

$$\delta W = \text{Vertical component of } \delta W_n = \delta W_n.\sin\alpha$$
$$= p_n \times 2\pi r.dr \text{ cosec } \alpha.\ \sin\alpha = p_n \times 2\pi\, r.dr$$

$\therefore$ Total vertical load transmitted to the bearing,

$$W = \int_0^R p_n \times 2\pi r.dr = 2\pi\, p_n \left[\frac{r^2}{2}\right]_0^R = 2\pi\, p_n \times \frac{R^2}{2} = \pi R^2.p_n$$

or $p_n = W / \pi R^2$

We know that frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W_n = \mu.p_n.2\pi r.dr \text{ cosec } \alpha = 2\pi\mu.p_n.\text{cosec } \alpha.r.dr$$

and frictional torque acting on the ring,

$$T_r = F_r \times r = 2\pi\mu.p_n.\text{cosec } \alpha.r.dr \times r = 2\pi\mu.p_n \text{ cosec } \alpha.r^2.dr$$

Integrating the expression within the limits from 0 to R for the total frictional torque on the conical pivot bearing.

$\therefore$ Total frictional torque,

$$T = \int_0^R 2\pi\mu.p_n \text{ cosec } \alpha.r^2\, dr = 2\pi\mu.p_n.\text{cosec } \alpha\left[\frac{r^3}{3}\right]_0^R$$

$$= 2\pi\mu.p_n.\text{cosec } \alpha \times \frac{R^3}{3} = \frac{2\pi R^3}{3} \times \mu.p_n.\text{cosec } \alpha \quad \text{...(i)}$$

Substituting the value of p_n in equation (i),

$$T = \frac{2\pi R^3}{3} \times \pi \times \frac{W}{\pi R^2} \times \text{cosec } \alpha = \frac{2}{3} \times \mu.W.R.\text{ cosec } \alpha$$

Consideringuniformwear

let$p_r$bethenormalintensityofpressureatadistance$r$fromthecentralaxis.

Weknowthat,incaseofuniformwear,theintensityofpressurevariesinverselywiththe distance

$\therefore \quad p_r r = C$ (a constant) or $p_r = C/r$

and the load transmitted to the ring,

$$\delta W = p_r \times 2\pi r.dr = \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

$\therefore$ Total load transmitted to the bearing,

$$W = \int_0^R 2\pi C.dr = 2\pi C\,[r]_0^R = 2\pi C.R \quad \text{or} \quad C = \frac{W}{2\pi R}$$

$$T_r = 2\pi\mu.p_r.\text{cosec } \alpha.r^2.dr = 2\pi\mu \times \frac{C}{r} \times \text{cosec } \alpha.r^2.dr$$
$$= 2\pi\mu.C.\text{cosec } \alpha.r.dr$$

$\therefore$ Total frictional torque acting on the bearing,

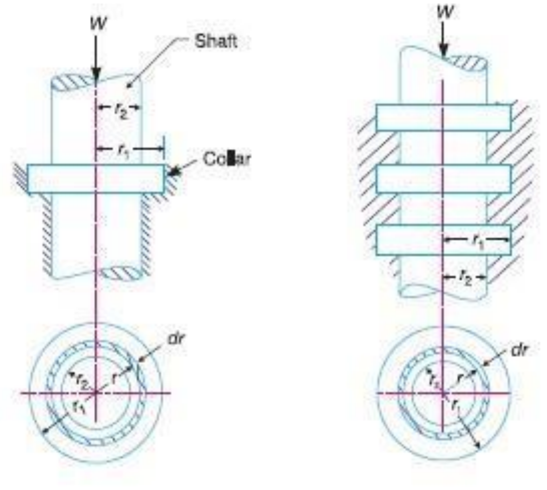
$$T = \int_0^R 2\pi\mu.C.\text{cosec } \alpha.r.dr = 2\pi\mu.C.\text{cosec } \alpha\left[\frac{r^2}{2}\right]_0^R$$
$$= 2\pi\mu.C.\text{cosec } \alpha \times \frac{R^2}{2} = \pi\mu.C.\text{cosec } \alpha.R^2$$

Substituting the value of C, we have

$$T = \pi\mu \times \frac{W}{2\pi R} \times \text{cosec } \alpha.R^2 = \frac{1}{2} \times \mu.W.R \text{ cosec } \alpha = \frac{1}{2} \times \mu.W.l$$

FlatCollarBearing

collar bearings are used to take the axial thrust of the rotating shafts. There may be a single collarormultiplecollarbearingsasshowninFig.Thecollarbearingsarealsoknownas ***thrust bearings.***



(*a*) Single collar bearing (*b*) Multiple collar bearing.

Consider a single flat collar bearing supporting a shaft as shown in Fig.

Let r_1 = External radius of the collar, and

r_2 = Internal radius of the collar.

$\therefore$ Area of the bearing surface,

$$A = \pi[(r_1)^2 - (r_2)^2]$$

Consideringuniformpressure

When the pressure is uniformly distributed over the bearing surface, then the intensity of pressure,

$$p = \frac{W}{A} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \qquad ...(i)$$

We have seen in Art. 10.25, that the frictional torque on the ring of radius r and thickness dr,

$$T_r = 2\pi\mu.p.r^2.dr$$

Integrating this equation within the limits from r_2 to r_1 for the total frictional torque on the collar.

$\therefore$ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\mu.p.r^2.dr = 2\pi\mu.p\left[\frac{r^3}{3}\right]_{r_2}^{r_1} = 2\pi\mu.p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p from equation (*i*),

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]}\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

Consideringunifromwear

$$\delta W = p_r.2\pi r.dr = \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

$\therefore$ Total load transmitted to the collar,

$$W = \int_{r_2}^{r_1} 2\pi C.dr = 2\pi C [r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

or
$$C = \frac{W}{2\pi(r_1 - r_2)} \quad ...(ii)$$

We also know that frictional torque on the ring,

$$T_r = \mu.\delta W.r = \mu \times 2\pi C.dr.r = 2\pi\mu.C.r.dr$$

$\therefore$ Total frictional torque on the bearing,

$$T = \int_{r_2}^{r_1} 2\pi\mu\, C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\mu.C[(r_1)^2 - (r_2)^2]$$

Substituting the value of C from equation *(ii)*,

$$T = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)}[(r_1)^2 - (r_2)^2] = \frac{1}{2} \times \mu.W(r_1 + r_2)$$

Module-3

Introduction

The power is transmitted from one shaft to the otherbymeans of belts, chains and gears. The belts and ropes are flexible members which are used where distance between the two shafts is large. The chains also have flexibility but they are preferred for intermediate distances. The gears are used when the shafts are very close with each other. This type of drive is also called positive drive because there is no slip. If the distance is slightly larger, chain drive can beused formaking it a positive drive. Belts and ropes transmit power due to the friction between the belt or rope and the pulley. There is a possibility of slip and creep and that is why, this drive is not a positive drive. A gear train is a combination of gears which are used for transmitting motion from one shaft to another.

PowerTransmissionDevices

Power transmission devices are very commonly used to transmit power from one shaft to another. Belts, chains and gears are used for this purpose. When the distance between the shafts is large, belts or ropes are used and for intermediate distance chains can be used. For belt drive distance can be maximum but this should not be more than ten metres for good results. Gear drive is used for short distances.

Belts

In case of belts, friction between the belt and pulley is used to transmit power. In practice, there is always some amount of slip between belt and pulleys, therefore, exact velocity ratio cannot be obtained. That is why, belt drive is not a positive drive. Therefore, the belt drive is used where exact velocity ratio is not required.

The flat belt is rectangular in cross-section. The pulley for this belt is slightly crowned to prevent slip of the belt to one side. It utilises the friction between the flat surface of the belt and pulley.

The V-belt is trapezoidal in section. It utilizes the force of friction between the inclined sides ofthe belt and pulley. They are preferred when distance is comparative shorter. Several V-belts can also be used together if power transmitted is more.

The circular belt or rope is circular in section Several ropes also can be used together to transmit more power.

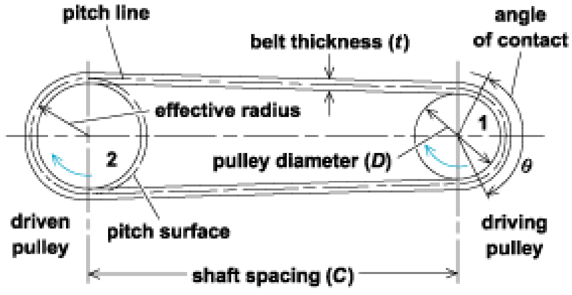
Thebeltdrivesareofthefollowingtypes:

(a) openbeltdrive,and

(b) crossbeltdrive.

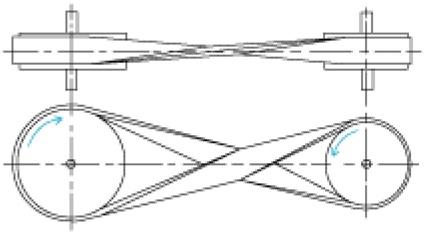
OpenBeltdrive

Open belt drive is used when sense of rotation of both the pulleys is same. It is desirable to keep the tight side of the belt on the lower side and slack side at the top to increase the angle of contact on the pulleys. This type of drive



CrossBeltDrive

In case of cross belt drive, the pulleys rotate in the opposite direction. The angle ofcontact of belt on both the pulleys is equal., the belt has to bend in two different planes.Asa result of this, belt wearsvery fastandtherefore, thistype of drive is not preferredfor power transmission. This can be used for transmission of speed atlow power.



Sincepowertransmittedbyabeltdriveisduetothefriction,beltdriveissubjectedtoslip and creep.

Velocityratioofbelt drive

Itisthe**ratiobetweenthevelocitiesofthedriverandthefolloweror driven.**

Let*d*1and*d*2bethediametersofdrivinganddrivenpulleys, respectively.

*N*1and*N*2bethecorrespondingspeedsofdrivinganddrivenpulleys,respectively. The velocity of the belt passing over the driver

$V_1=\pi d_1N_1/60$

Ifthereisnoslipbetweenthebeltandp

$V_1=V_2=\pi d_2N_2/60$

Or, $\pi d_1N_1/60=\pi d_2N_2/60$

Or, $N_1/N_2=d_2/d_1$

Ifthicknessofthebeltis'*t*',anditisnotnegligibleincomparisontothe diameter,

Or , $N_1/N_2=d_2+t/d_1+t$

Lettherebetotalpercentageslip'*S*'inthebeltdrivewhichcanbetakenintoaccountas follows

$V_2=V_1(1-S/100)$

$\pi d_2N_2/60=\pi d_1N_1/60(1-S/100)$

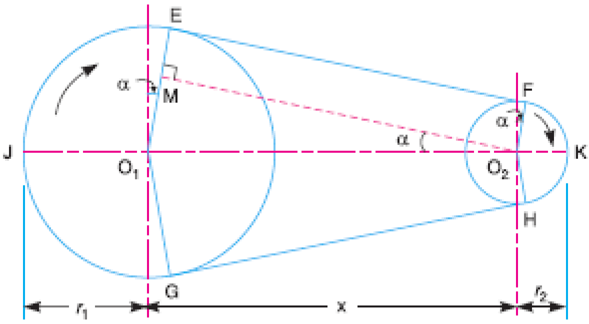
Ifthethicknessofbeltisalsotobeconsidered

, $N_1/N_2=d_2+t/d_1+t \quad *1/(1-S/100)$

$N_2/ N_1= d_1+t \quad / d_2+t *(1-S/100)$

The belt moves from the tight side to the slack side and vice-versa, there is some loss of powerbecausethelengthofbeltcontinuouslyextendsontightsideandcontractsonloose side. Thus, there is relative motion between the belt and pulley due to body slip. This is known **as creep.**

Lengthofopenbeltdrive



We have already discussed in Art. 11.6 that in an open belt drive, both the pulleys rotate in the *same* direction as shown in Fig. 11.11.

Let r_1 and r_2 = Radii of the larger and smaller pulleys,

x = Distance between the centres of two pulleys (*i.e.* $O_1\,O_2$), and

L = Total length of the belt.

Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H as shown in Fig. 11.11. Through O_2, draw $O_2\,M$ parallel to FE.

From the geometry of the figure, we find that $O_2\,M$ will be perpendicular to $O_1\,E$.

Let the angle $M\,O_2\,O_1$ = α radians.

We know that the length of the belt,

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$

$$= 2\,(\text{Arc } JE + EF + \text{Arc } FK) \qquad \text{...(}i$$

From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_1\,M}{O_1\,O_2} = \frac{O_1\,E - EM}{O_1\,O_2} = \frac{r_1 - r_2}{x}$$

Since α is very small, therefore putting

$$\sin\alpha = \alpha \text{ (in radians)} = \frac{r_1 - r_2}{x} \qquad \text{...(}ii$$

$$\therefore \qquad \text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right) \qquad \text{...(}iii$$

Similarly $$\text{Arc } FK = r_2\left(\frac{\pi}{2} - \alpha\right) \qquad \text{...(}iv$$

and $$EF = MO_2 = \sqrt{(O_1\,O_2)^2 - (O_1\,M)^2} = \sqrt{x^2 - (r_1 - r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem,

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2 + \ldots\right] = x - \frac{(r_1 - r_2)^2}{2x} \qquad \text{...(}v\text{)}$$

Substituting the values of arc JE from equation (*iii*), arc FK from equation (*iv*) and EF from equation (*v*) in equation (*i*), we get

$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 - r_2)^2}{2x} + r_2\left(\frac{\pi}{2} - \alpha\right)\right]$$

$$= 2\left[r_1 \times \frac{\pi}{2} + r_1.\alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2 \times \frac{\pi}{2} - r_2.\alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

Substituting the value of $\alpha = \frac{r_1 - r_2}{x}$ from equation (*ii*),

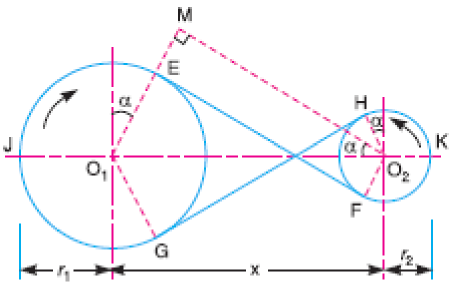
$$L = \pi(r_1 + r_2) + 2 \times \frac{(r_1 - r_2)}{x} \times (r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{2(r_1 - r_2)^2}{x} + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x} \qquad \text{...(In terms of pulley radii)}$$

$$= \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x} \qquad \text{...(In terms of pulley diameters)}$$

LengthofaCrossBeltDrive



Let r_1 and r_2 = Radii of the larger and smaller pulleys,

x = Distance between the centres of two pulleys (*i.e.* $O_1 O_2$), and

L = Total length of the belt.

Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H, as shown in Fig. 11.12. Through O_2, draw O_2M parallel to FE.

From the geometry of the figure, we find that O_2M will be perpendicular to O_1E.

Let the angle $MO_2\,O_1 = \alpha$ radians.

We know that the length of the belt,

$$L = \text{Arc } GJE + EF + \text{Arc } FKH + HG$$

$$= 2\,(\text{Arc } JE + EF + \text{Arc } FK) \qquad \text{...}(i)$$

From the geometry of the figure, we find that

$$\sin\alpha = \frac{O_1 M}{O_1 O_2} = \frac{O_1 E + EM}{O_1 O_2} = \frac{r_1 + r_2}{x}$$

Since α is very small, therefore putting

$$\sin\alpha = \alpha \text{ (in radians)} = \frac{r_1 + r_2}{x} \qquad \text{...}(ii)$$

$$\therefore \qquad \text{Arc } JE = r_1\left(\frac{\pi}{2} + \alpha\right) \qquad \text{...}(iii)$$

Similarly

$$\text{Arc } FK = r_2\left(\frac{\pi}{2} + \alpha\right) \qquad \text{...}(iv)$$

and

$$EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{x^2 - (r_1 + r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 + r_2}{x}\right)^2}$$

Expanding this equation by binomial theorem,

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 + r_2}{x}\right)^2 + \ldots\right] = x - \frac{(r_1 + r_2)^2}{2x} \qquad \ldots(v)$$

Substituting the values of arc JE from equation *(iii)*, arc FK from equation *(iv)* and EF from equation *(v)* in equation *(i)*, we get

$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 + r_2)^2}{2x} + r_2\left(\frac{\pi}{2} + \alpha\right)\right]$$

$$= 2\left[r_1 \times \frac{\pi}{2} + r_1.\alpha + x - \frac{(r_1 + r_2)^2}{2x} + r_2 \times \frac{\pi}{2} + r_2.\alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 + r_2) + x - \frac{(r_1 + r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

Substituting the value of $\alpha = \frac{r_1 + r_2}{x}$ from equation *(ii)*,

$$L = \pi(r_1 + r_2) + \frac{2(r_1 + r_2)}{x} \times (r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{2(r_1 + r_2)^2}{x} + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x} \qquad \ldots\text{(In terms of pulley radii)}$$

$$= \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x} \qquad \ldots\text{(In terms of pulley diameters)}$$

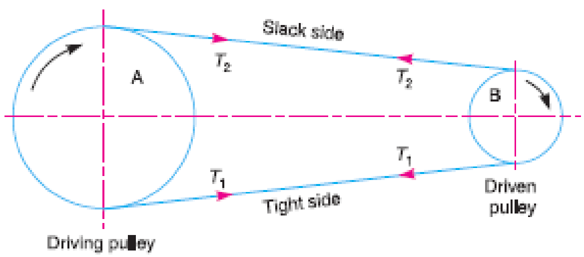
PowerTransmittedbyaBelt

thedrivingpulley(ordriver) *A*andthedrivenpulley(orfollower) *B*.We havealreadydiscussedthatthedrivingpulleypullsthebeltfromonesideanddelivers the same to the otherside.Itisthusobviousthatthetensionontheformerside(*i.e*.tightside)willbe greater than the latterside(*i.e*.slackside)

T_1andT_2=Tensionsinthetightandslacksideofthebeltrespectivelyinnewtons,

$r_1$andr_2=Radiiofthedriverandfollowerrespectively,and

v=Velocityofthebeltin m/s.

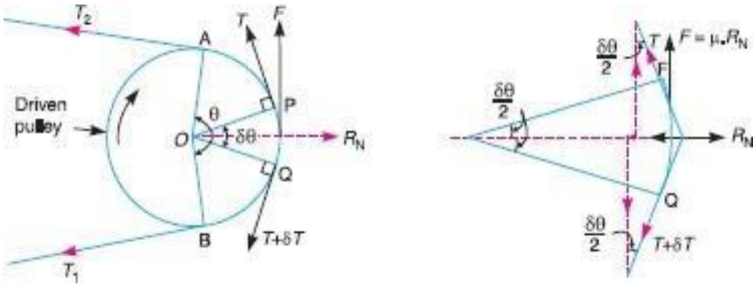


The effective turning (driving) force at the circumference of the follower is the difference between the two tensions (*i.e.* $T_1 - T_2$).

$\therefore$ Work done per second $= (T_1 - T_2)\, v$ N-m/s

and power transmitted, $P = (T_1 - T_2)\, v$ W ...($\because$ 1 N-m/s = 1 W)

RatioofDrivingTensionsForFlatBelt Drive



T_1=Tensioninthebeltonthetightside,

T_2=Tensioninthebeltontheslackside, and

θ=Angleofcontactinradians(*i.e.*anglesubtendedby thearc *AB*,alongwhichthe belt touches the pulley at the centre).

Nowconsiderasmallportionofthebelt *PQ*,subtendingananglеθatthecentreofthe pulley .The belt *PQ* is in equilibrium under the following forces

1. Tension T in the belt at P,
2. Tension $(T + \delta T)$ in the belt at Q,
3. Normal reaction R_N, and
4. Frictional force, $F = \mu \times R_N$, where μ is the coefficient of friction between the belt and pulley.

Resolving all the forces horizontally and equating the same,

$$R_N = (T + \delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2} \qquad ...(i)$$

Since the angle $\delta\theta$ is very small, therefore putting $\sin \delta\theta / 2 = \delta\theta / 2$ in equation (*i*),

$$R_N = (T + \delta T)\frac{\delta\theta}{2} + T \times \frac{\delta\theta}{2} = \frac{T.\delta\theta}{2} + \frac{\delta T.\delta\theta}{2} + \frac{T.\delta\theta}{2} = T.\delta\theta \qquad ...(ii)$$

$$...\left(\text{Neglecting } \frac{\delta T.\delta\theta}{2}\right)$$

Now resolving the forces vertically, we have

$$\mu \times R_N = (T + \delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2} \quad \text{...(iii)}$$

Since the angle $\delta\theta$ is very small, therefore putting $\cos \delta\theta / 2 = 1$ in equation *(iii)*,

$$\mu \times R_N = T + \delta T - T = \delta T \text{ or } R_N = \frac{\delta T}{\mu} \quad \text{...(iv)}$$

Equating the values of R_N from equations *(ii)* and *(iv)*,

$$T.\delta\theta = \frac{\delta T}{\mu} \text{ or } \frac{\delta T}{T} = \mu.\delta\theta$$

Integrating both sides between the limits T_2 and T_1 and from 0 to θ respectively,

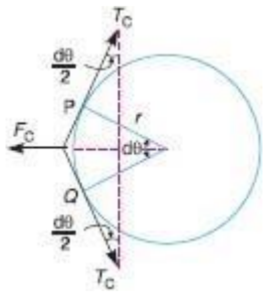
i.e. $$\int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_0^{\theta} \delta\theta \quad \text{or} \quad \log_e\left(\frac{T_1}{T_2}\right) = \mu.\theta \text{ or } \frac{T_1}{T_2} = e^{\mu.\theta} \quad \text{...(v)}$$

Equation *(v)* can be expressed in terms of corresponding logarithm to the base 10, *i.e.*

$$2.3\log\left(\frac{T_1}{T_2}\right) = \mu.\theta$$

CentrifugalTension

thebeltcontinuouslyruns overthepulleys,therefore,somecentrifugalforceiscaused, whose effect is to increase the tension on both, tight as well as the slack sides. The tension causedby centrifugal forceis called***centrifugal tension***. At lowerbelt speeds (less than 10 m/s), the centrifugal tension is very small, but at higher beltspeeds (more than 10 m/s), its effectis considerable and thus should be taken intoaccount.



Let m = Mass of the belt per unit length in kg,

v = Linear velocity of the belt in m/s,

r = Radius of the pulley over which the belt runs in metres, and

T_C = Centrifugal tension acting tangentially at P and Q in newtons.

We know that length of the belt PQ

$$= r.\,d\theta$$

and mass of the belt PQ $= m.\,r.\,d\theta$

$\therefore$ Centrifugal force acting on the belt PQ,

$$F_C = (m.r.d\theta)\frac{v^2}{r} = m.d\theta.v^2$$

The centrifugal tension T_C acting tangentially at P and Q keeps the belt in equilibrium.

Now resolving the forces (*i.e.* centrifugal force and centrifugal tension) horizontally and equating the same, we have

$$T_C \sin\left(\frac{d\theta}{2}\right) + T_C \sin\left(\frac{d\theta}{2}\right) = F_C = m.d\theta.v^2$$

Since the angle $d\theta$ is very small, therefore, putting $\sin\left(\frac{d\theta}{2}\right) = \frac{d\theta}{2}$, in the above expression,

$$2T_C\left(\frac{d\theta}{2}\right) = m.d\theta.v^2 \quad \text{or} \quad T_C = m\,.\,v^2$$

MaximumTensionintheBelt

A little consideration will show that the maximum tension in the belt (T) is equal to the total tension in the tight side of the belt (T_{t1}).

Let σ = Maximum safe stress in N/mm^2,

b = Width of the belt in mm, and

t = Thickness of the belt in mm.

We know that maximum tension in the belt,

T = Maximum stress × cross-sectional area of belt = $\sigma.\,b.\,t$

When centrifugal tension is neglected, then

T (or T_{t1}) = T_1, *i.e.* Tension in the tight side of the belt

and when centrifugal tension is considered, then

$$T \text{ (or } T_{t1}) = T_1 + T_C$$

ConditionFortheTransmissionofMaximumPower

We know that power transmitted by a belt,

$$P = (T_1 - T_2)\,v \qquad ...(i)$$

where T_1 = Tension in the tight side of the belt in newtons,

T_2 = Tension in the slack side of the belt in newtons, and

v = Velocity of the belt in m/s.

From Art. 11.14, we have also seen that the ratio of driving tensions is

$$\frac{T_1}{T_2} = e^{\mu.\theta} \quad \text{or} \quad T_2 = \frac{T_1}{e^{\mu.\theta}} \qquad ...(ii)$$

Substituting the value of T_2 in equation (*i*),

$$P = \left(T_1 - \frac{T_1}{e^{\mu.\theta}}\right)v = T_1\left(1 - \frac{1}{e^{\mu.\theta}}\right)v = T_1.v.C \qquad ...(iii)$$

where $\quad C = 1 - \frac{1}{e^{\mu.\theta}}$

We know that $\quad T_1 = T - T_C$

where $\quad T$ = Maximum tension to which the belt can be subjected in newtons, and

T_C = Centrifugal tension in newtons.

Substituting the value of T_1 in equation *(iii)*,

$$P = (T - T_C)\, v.C$$

$$= (T - m.v^2)\, v.C = (T.v - m\, v^3)\, C \quad \text{... (Substituting } T_C = m.\, v^2\text{)}$$

For maximum power, differentiate the above expression with respect to v and equate to zero, *i.e.*

$$\frac{dP}{dv} = 0 \quad \text{or} \quad \frac{d}{dv}(T.v - mv^3)C = 0$$

$\therefore \quad T - 3\, m\, .\, v^2 = 0$

or $\quad T - 3\, T_C = 0$ or $T = 3\, T_C$ ...*(iv)*

It shows that when the power transmitted is maximum, 1/3rd of the maximum tension is absorbed as centrifugal tension.

InitialTensionintheBelt

Whenabeltiswoundroundthetwopulleys(*i.e*.driverandfollower),itstwoends are joined together;sothatthebeltmaycontinuouslymoveoverthepulleys,sincethemotionof the belt from

thedriverandthefollower isgovernedbyafirmgrip,duetofrictionbetweenthebelt and the pulleys.

Inorder toincreasethisgrip,thebeltistightenedup. Atthisstage,evenwhenthe pulleys are stationary,

thebeltissubjectedtosometension,called***initialtension***.

Whenthedriverstartsrotating,it pullsthebeltfromoneside(increasingtensioninthe belt onthisside)anddelivers ittotheotherside(decreasingthetensioninthebeltonthat side). The

increasedtensioninonesideofthebeltiscalledtensionintightsideandthe decreased tension in the

othersideofthebeltiscalledtensionintheslack side.

Let T_0 = Initial tension in the belt,

T_1 = Tension in the tight side of the belt,

T_2 = Tension in the slack side of the belt, and

α = Coefficient of increase of the belt length per unit force.

A little consideration will show that the increase of tension in the tight side

$$= T_1 - T_0$$

and increase in the length of the belt on the tight side

$$= \alpha (T_1 - T_0) \quad ...(i)$$

Similarly, decrease in tension in the slack side

$$= T_0 - T_2$$

and decrease in the length of the belt on the slack side

$$= \alpha (T_0 - T_2) \quad ...(ii)$$

Assuming that the belt material is perfectly elastic such that the length of the belt remains constant, when it is at rest or in motion, therefore increase in length on the tight side is equal to decrease in the length on the slack side. Thus, equating equations (*i*) and (*ii*),

$$\alpha (T_1 - T_0) = \alpha (T_0 - T_2) \quad \text{or} \quad T_1 - T_0 = T_0 - T_2$$

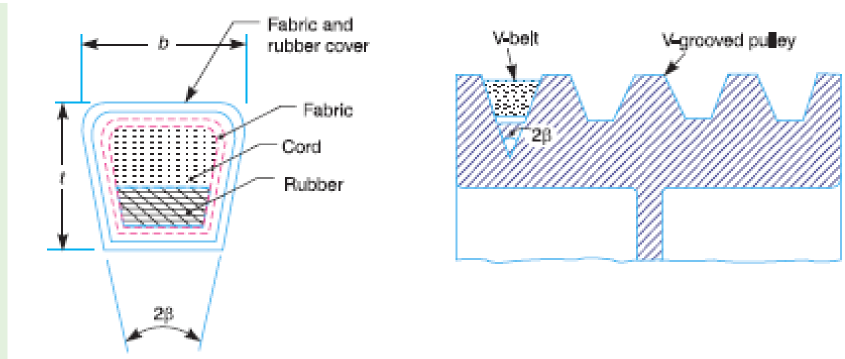
$$\therefore \quad T_0 = \frac{T_1 + T_2}{2} \quad \text{...(Neglecting centrifugal tension)}$$

$$= \frac{T_1 + T_2 + 2T_C}{2} \quad \text{...(Considering centrifugal tension)}$$

V-beltdrive

V-belt is mostly used in factories and workshops where a great amount of power is to be transmitted from one pulley to another when the two pulleys are very near to each other

The V-belts are made of fabric and cords moulded in rubber and covered with fabric and rubber,Thesebeltsaremouldedtoatrapezoidalshapeand aremadeendless.These are particularly suitable for short drives *i.e.* when the shafts are at a short distance apart. The included angle for the V-belt is usually from 30° – 40°. In case of flat belt drive, the belt runs over the pulleys whereas in case of V-belt drive, the rim of the pulley is grooved in which the V-belt runs. The effect of the groove is to increase the frictionalgripoftheV-beltonthepulleyandthustoreducethetendency ofslipping.In order to have a good grip on the pulley,the V-belt is in contact with the side faces of the groove and not at the bottom. The power is transmitted by the *wedging action between the belt and the V-groove in the pulley



(*a*) Cross-section of a V-belt. (*b*) Cross-section of a V-grooved pulley.

Gears

Gears are also used for power transmission. This is accomplished by the successive engagementofteeth.Thetwo gears transmitmotion by the directcontactlikechaindrive. Gears also provide positive drive.

The drive between the two gears can be represented by using plain cylinders or discs 1and 2 having diameters equal to their pitch circles as shown in Figure 3.5. The point of contact of the two pitch surfaces shell have velocity along the common tangent. Because there is no slip, definite motion of gear 1 can be transmitted to gear 2 or vice-versa.

Thetangentialvelocity‘V_p’$=\omega_1 r1$

$=\omega 2r2$

where$r$1and$r$2arepitchcircleradiiofgears1and2, respectively

Gear

Gearsarealsousedforpowertransmission.Thisisaccomplishedbythesuccessive engagement of teeth.They can be applied between two shafts which are

Parallel

Collinear

Perpendicular and intersecting

Perpendicularandnonintersecting

Inclined at any arbitrary angle

Classifygears

***Accordingtothepositionofaxesoftheshafts*.**Theaxesofthetwoshaftsbetweenwhich the motion is to be transmitted, may be

Parallel,**(*b*)**Intersecting,and**(*c*)**Non-intersectingandnon-parallel.

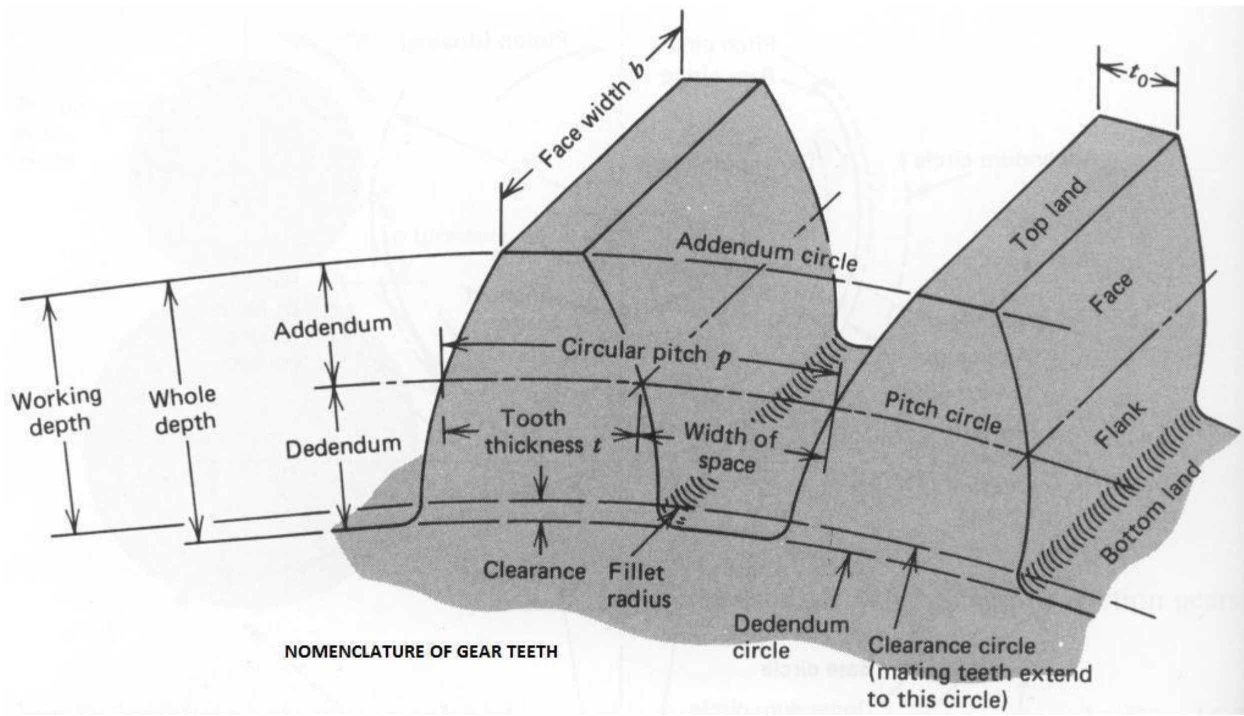
2. *Accordingtotheperipheralvelocityofthegears*.Thegears,accordingtothe peripheral

velocityofthegearsmaybeclassifiedas:

(*a*)Lowvelocity,**(*b*)**Mediumvelocity,and**(*c*)**Highvelocity

Accordingtothetypeofgearing.Thegears,accordingtothetypeofgearingmaybe classified as : Externalgearing,**(*b*)**Internalgearing,and**(*c*)**Rackand pinion.

Accordingtopositionofteethonthegearsurface.Theteethonthegearsurfacemay be **(*a*)**straight,**(*b*)**inclined,and**(*c*)**curved



NOMENCLATURE OF GEAR TEETH

Pitchcircle.Itisanimaginarycirclewhichbypurerollingaction,wouldgivethesame motion as the actual gear.

2. ***Pitchcirclediameter***.Itisthediameterofthepitchcircle.Thesizeofthegearis usually specified by the pitch circle diameter. It is also known as***pitchdiameter***.

3. ***Pitchpoint***.Itisacommonpointofcontactbetweentwopitchcircles.

4. ***Pitchsurface***.Itisthesurfaceoftherollingdiscswhichthemeshinggearshave replaced at the pitch circle.

5. ***Pressure angle or angle of obliquity***. It is the angle between the common normal to two gear teethatthepointof contactand thecommon tangentatthepitch point. The standard pressure angles are 14□□and 20°.

6. ***Addendum*.**Itistheradialdistanceofatoothfromthepitchcircletothetopofthe tooth.

7. ***Dedendum*.**Itistheradialdistanceofatoothfromthepitchcircletothebottomofthe tooth.

8. ***Addendumcircle***.Itisthecircledrawnthroughthetopoftheteethandisconcentric with the pitch circle.

9. ***Dedendumcircle***.Itisthecircledrawnthroughthebottomof theteeth.Itis alsocalled root circle.

10. ***Circular pitch***.

Itisthedistancemeasuredonthecircumferenceofthepitch circlefromapointofonetoothtothecorrespondingpointonthenexttooth.Itisusually denoted by *pc*.

Mathematically,

Circularpitch,=πD/T

Where, D=diameterofpitchcircle

T=numberofteethonthewheel ***Diametral pitch.*** It is the ratio of number of teeth to the pitch circle diameter in millimetres.

Itisdenotedby*pd*

. Mathematically,

Diametralpitch,P_d=T/D

, D=diameterofpitchcircle

T=numberofteethonthe wheel

***Module*.**Itistheratioofthepitchcirclediameterinmillimeterstothenumberofteeth. It is usually denoted by *m*. Mathematically,

m=D/T

Working depth. It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

16. ***Ooththickness***.Itisthewidthofthetoothmeasuredalongthepitchcircle.

17. ***Tooth space*** . It is the width of space between the two adjacent teeth measured along the pitch circle.

18. ***Backlash***. It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.

19. ***Faceoftooth***.Itisthesurfaceofthegeartoothabovethepitchsurface.

20. ***Flankoftooth***.Itisthesurfaceofthegeartoothbelowthepitchsurface.

21. ***Topland***.Itisthesurfaceofthetopofthetooth.

22. ***Facewidth***.Itisthewidthofthegeartoothmeasuredparalleltoitsaxis.

23. ***Profile***.Itisthecurveformedbythefaceandflankofthetooth.

24. ***Filletradius***.Itistheradiusthatconnectstherootcircletotheprofileofthetooth.

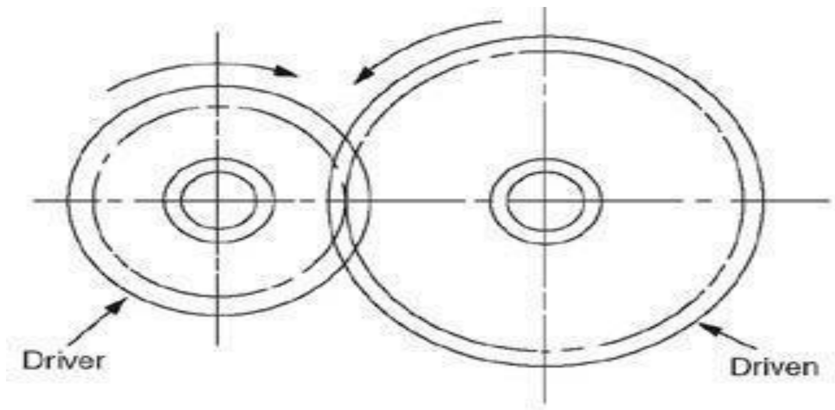
25. ***Path of contact***. It is the path traced by the point of contact of two teeth from the beginning to the end ofengagement.

26. ****Length of the path of contact***. It is the length of the common normal cut-off by the addendum circles of the wheel andpinion.

27. ** ***Arc of contact***. It is the path traced by a point on the pitch circle from the beginning to the end ofengagement of agivenpair of teeth. Thearc ofcontactconsists of two parts, *i.e*.

(*a*) ***Arc of approach***. It is the portion of the path of contact from the beginning of the engagement to the pitch point

Simplegear train

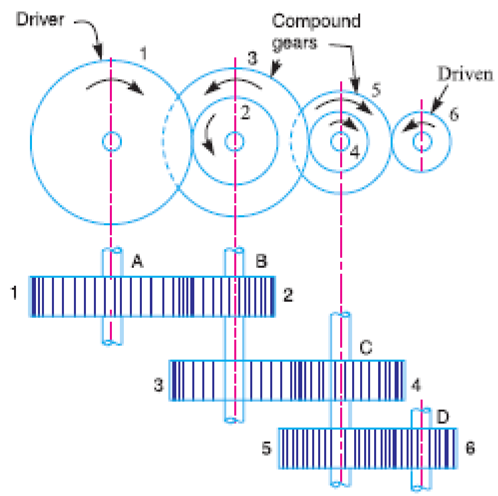


Asimplegeartrainusestwogears,whichmaybeofdifferentsizes.Ifoneofthesegearsis attached to a motor or a crank then it is called the driver gear.The gear that is turned by the driver gear is called the driven gear. The input and the output shaft are necessarily being parallel to each other. In this gear train, there are series of gears which are capable of receivingand transmitting motion from one gear to another. They may mesh externally or internally. Each gear rotates about separate axis fixed to the frame. Two gears may be external meshing and internal meshing.

Velocityratio:

$N_1/N_2=T_2/T_1=d_2/d_1$

Compoundgeartrain



Whentherearemorethanonegearonashaft,,itiscalleda***compoundtrainofgear.***
inasimpletrainofgearsdonoteffectthespeedratioofthesystem.Butthesegearsareuseful in bridging overthe space between the driver and the driven

In a compound train of gears, as shown in Fig .the gear 1 is the driving gear mounted on shaft*A*,gears2and3arecompoundgearswhicharemountedonshaft*B*.Thegears4and5 are also compoundgearswhicharemountedonshaft*C*andthegear6isthedrivengearmounted on shaft *D*.

Let N_1 = Speed of driving gear 1,
T_1 = Number of teeth on driving gear 1,
$N_2, N_3 ..., N_6$ = Speed of respective gears in r.p.m., and
$T_2, T_3 ..., T_6$ = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad ...(i)$$

Similarly, for gears 3 and 4, speed ratio is

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \quad ...(ii)$$

and for gears 5 and 6, speed ratio is

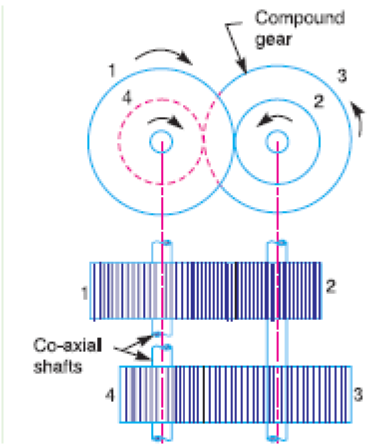
$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \quad ...(iii)$$

The speed ratio of compound gear train is obtained by multiplying the equations (*i*), (*ii*) and (*iii*),

$$\therefore \quad \frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5} \quad \text{or} \quad {}^*\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

Revertedgeartrain

When the axes of the first gear (*i.e.* first driver) and the last gear (*i.e.* last driven or follower) are co-axial, then the gear train is known as ***reverted gear* train** as shown in Fig.
We see that gear 1 (*i.e.* first driver) drives the gear 2 (*i.e.* first driven or follower) in the opposite direction. Since the gears 2 and 3 are mounted on the same shaft, therefore they forma compound gear andthe gear 3 willrotate in the same direction as that of gear 2. The gear3(whichisnowtheseconddriver)drivesthegear4(*i.e.*thelastdrivenorfollower)inthe same direction as that of gear 1. Thus wesee that in a revertedgear train, the motion of the first gear and the last gear is ***like.***



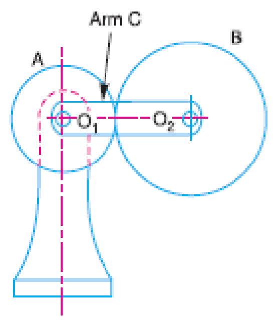
$$\text{Speed ratio} = \frac{\text{Product of number of teeth on drivens}}{\text{Product of number of teeth on drivers}}$$

$$\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3}$$

Epicyclicgeartrain

inanepicyclic gear train,the axesoftheshafts, overwhichthegearsaremounted,may move relative to a fixed axis. A simple epicyclic gear train is shown in Fig., wherea gear *A* and the

armCthaveacommonaxisatO_1aboutwhichtheycanrotate.ThegearBmesheswithgearA and has its axis on the arm at O_2, about which the gear *B* can rotate. If the arm is fixed, the gear train is simple and gear *A* can drive gear *B*
or ***vice- versa***, but if gear *A* is fixed and the arm is rotated about the axis of gear *A* (*i.e.* O_1), thenthegear*B*isforcedtorotate***upon***and***around***gear*A*.Suchamotioniscalled**epicyclic** and the gear trains arrangedin such a manner that oneor more oftheir members moveupon andaroundanothermemberareknownas***epicyclicgeartrains***(***epi*.**meansuponand***cyclic*** means around). The epicyclic gear trains may be ***simple*** or ***compound*.** The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space. The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.



Module-4

Governors and Flywheel 4.1 Function of governor 4.2 Classification of governor 4.3 Working of Watt,Porter,ProelandHartnellgovernors.4.4Conceptualexplanationofsensitivity,stability and isochronisms. 4.5 Function of flywheel. 4.6 Comparison between flywheel &governor. 4.7 Fluctuationofenergyandcoefficientoffluctuationofspeed.

GOVERNORANDFLYWHEEL

Introduction

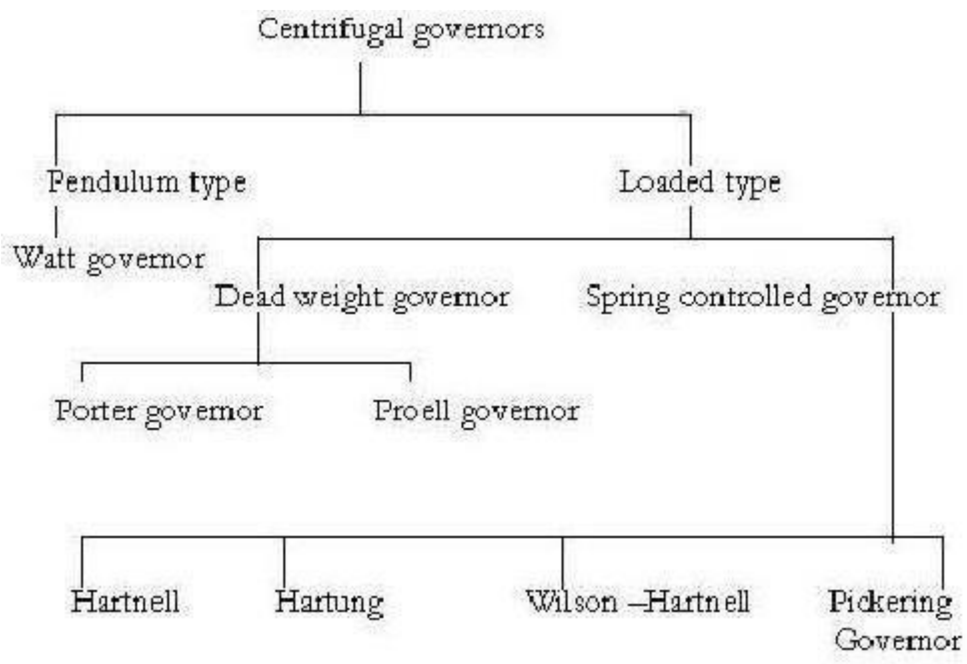
Thegovernor is a devicewhich isregulate the mean speed of an engine, whenthere are variations in the load, during long periods. On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required.the governor has no influence over cyclic speed fluctuation.

Typesofgovernor

Governors are classified based upon two different principles. These are:Centrifugal governors are further classified as –

- Centrifugalgovernor
- Inertiagovernors

Centrifugalgovernorsarefurtherclassifiedas–



WattGovernor

Thesimplestformofacentrifugalgovernorisa Wattgovernor,Itconsistofpairof twoballsand which is attached with the spindle with help of arms.The upper arm pinned at point O. the lower arm arefixed connect to the sleeve. The sleeve freely move on the spindle which is driven by engine. The spindle rotate the balls take of position depending upon speed ofspindle

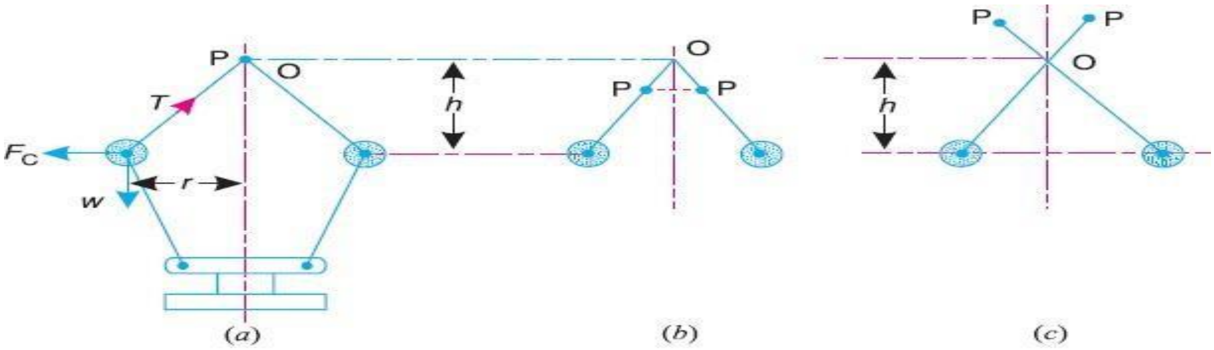
Thearmsofthegovernormaybeconnectedtothespindleinthefollowingthreeways:

1. ThepivotP,maybeonthespindleaxis.

2. ThepivotP,maybeoffsetfromthespindleaxisandthe armswhenproducedintersectatO.

3. ThepivotP,maybeoffset,butthearmscrosstheaxisatO.

Let



Watt governor.

m=Massoftheballinkg,

w=Weightoftheballinnewtons= $m.g$, T =

Tension in the arm in newtons,

ω=Angularvelocityofthearmandballaboutthespindleaxisin rad/s,

r=Radiusofthepathofrotationoftheball *i.e.*horizontaldistancefromthecentre oftheballtothe spindle axis in metres,

F_C=Centrifugalforceactingontheballinnewtons=mv^2/r

h=Heightofthegovernorinmetres.

Itisassumedthattheweightofthearms,linksandthesleevearenegligibleascomparedto the weight of the balls. Now, the ball is in equilibrium under the action of

1. thecentrifugalforce(F_C)actingontheball,

2. thetension(T)inthearm,

3. theweight(w)oftheball.

Taking moments about point O, we have

$$F_C \times h = w \times r = m.g.r$$

or $$m.\omega^2.r.h = m.g.r \quad \text{or} \quad h = g/\omega^2 \qquad \ldots (i)$$

When g is expressed in m/s² and ω in rad/s, then h is in metres. If N is the speed in r.p.m., then

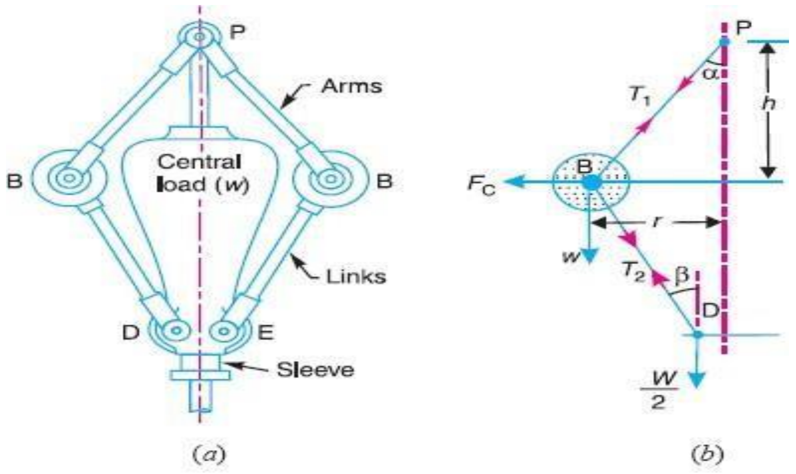
$$\omega = 2\pi N/60$$

∴ $$h = \frac{9.81}{(2\pi N/60)^2} = \frac{895}{N^2} \text{ metres} \qquad \ldots (\because g = 9.81 \text{ m/s}^2) \ldots (ii)$$

PorterGovernor

It differs from the watt governor is in the use of a heavily weighted sleeve. The load moves up and down the central spindle. This additional downward force increases the speed of revolution required to enable the balls to rise to any predetermined level.

Considertheforcesactingonone-halfofthegovernor



Porter governor.

Let$m$=Massofeachballinkg,
w=Weightofeachballinnewtons= $m.g$, M =
Mass of the central load in kg,
W=Weightofthecentralloadinnewtons= $M.g$, r =
Radius of rotation in metres,
h=Heightofgovernorinmetres,
N=Speedoftheballsinr.p.m .,
ω=Angularspeedoftheballsinrad/s=$2\pi N/60$ rad/s,
F_C=Centrifugalforceactingontheballinnewtons=mv^2/r, T_1 =
Force in the arm in newtons,
T_2=Forceinthelinkin newtons,
α=Angleofinclinationofthearm(orupperlink)tothevertical,and
β=Angleofinclinationofthelink(orlowerlink)tothevertical.
Theweightofarmsandweightofsuspensionlinksandeffectoffrictiontothemovementofsleeveare neglected.
Thoughthereareseveralwaysofdeterminingtherelationbetweentheheightofthegovernor(h)and the angular speed of the balls (ω).

1. *Method of resolution of forces*

Considering the equilibrium of the forces acting at D, we have

$$T_2 \cos\beta = \frac{W}{2} = \frac{M.g}{2}$$

or

$$T_2 = \frac{M.g}{2\cos\beta}$$

Again, considering the equilibrium of the forces acting on B. The point B is in equilibrium under the action of the following forces, as shown in Fig. 18.3 (*b*).

(*i*) The weight of ball ($w = m.g$),

(*ii*) The centrifugal force (F_C),

(*iii*) The tension in the arm (T_1), and

(*iv*) The tension in the link (T_2).

Resolving the forces vertically,

$$T_1 \cos\alpha = T_2 \cos\beta + w = \frac{M.g}{2} + m.g \qquad \ldots (ii)$$

$$\ldots \left(\because T_2 \cos\beta = \frac{M.g}{2} \right)$$

Resolving the forces horizontally,

$$T_1 \sin\alpha + T_2 \sin\beta = F_C$$

$$T_1 \sin\alpha + \frac{M.g}{2\cos\beta} \times \sin\beta = F_C \qquad \ldots \left(\because T_2 = \frac{M.g}{2\cos\beta} \right)$$

$$T_1 \sin\alpha + \frac{M.g}{2} \times \tan\beta = F_C$$

$$\therefore \qquad T_1 \sin\alpha = F_C - \frac{M.g}{2} \times \tan\beta \qquad \ldots (iii)$$

Dividing equation (*iii*) by equation (*ii*),

$$\frac{T_1 \sin\alpha}{T_1 \cos\alpha} = \frac{F_C - \frac{M.g}{2} \times \tan\beta}{\frac{M.g}{2} + m.g}$$

or

$$\left(\frac{M.g}{2} + m.g \right) \tan\alpha = F_C - \frac{M.g}{2} \times \tan\beta$$

$$\frac{M.g}{2} + m.g = \frac{F_C}{\tan\alpha} - \frac{M.g}{2} \times \frac{\tan\beta}{\tan\alpha}$$

Substituting $\frac{\tan\beta}{\tan\alpha} = q$, and $\tan\alpha = \frac{r}{h}$, we have

$$\frac{M.g}{2} + m.g = m.\omega^2.r \times \frac{h}{r} - \frac{M.g}{2} \times q \qquad \ldots (\because F_C = m.\omega^2.r)$$

or

$$m.\omega^2.h = m.g + \frac{M.g}{2}(1+q)$$

$$\therefore \qquad h = \left[m.g + \frac{M.g}{2}(1+q) \right] \frac{1}{m.\omega^2} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{\omega^2} \qquad \ldots (iv)$$

or

$$\omega^2 = \left[m.g + \frac{Mg}{2}(1+q) \right] \frac{1}{m.h} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}$$

or

$$\left(\frac{2\pi N}{60} \right)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}$$

$$\therefore \qquad N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h} \left(\frac{60}{2\pi} \right)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h}$$

2. *Instantaneous centre method*

In this method, equilibrium of the forces acting on the link BD are considered. The instantaneous centre I lies at the point of intersection of PB produced and a line through D perpendicular to the spindle axis, as shown in Fig. 18.4. Taking moments about the point I,

$$F_C \times BM = w \times IM + \frac{W}{2} \times ID$$

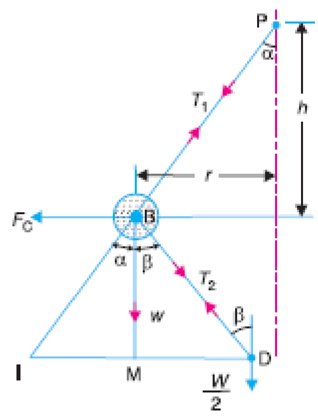
$$= m.g \times IM + \frac{M.g}{2} \times ID$$

$$\therefore \quad F_C = m.g \times \frac{IM}{BM} + \frac{M.g}{2} \times \frac{ID}{BM}$$

$$= m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM + MD}{BM}\right)$$

$$= m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM}{BM} + \frac{MD}{BM}\right)$$

$$= m.g \tan\alpha + \frac{M.g}{2}(\tan\alpha + \tan\beta)$$



Dividing throughout by tan α,

$$\frac{F_C}{\tan\alpha} = m.g + \frac{M.g}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right) = m.g + \frac{M.g}{2}(1+q) \qquad \ldots\left(\because q = \frac{\tan\beta}{\tan\alpha}\right)$$

We know that $F_C = m.\omega^2.r$, and $\tan\alpha = \frac{r}{h}$

$$\therefore \quad m.\omega^2.r \times \frac{h}{r} = m.g + \frac{M.g}{2}(1+q)$$

or

$$h = \frac{m.g + \frac{M.g}{2}(1+q)}{m} \times \frac{1}{\omega^2} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{\omega^2}$$

When $\tan\alpha = \tan\beta$ or $q = 1$, then

$$h = \frac{m + M}{m} \times \frac{g}{\omega^2}$$

ProellGovernor

The proell governor has ball fixed at B and C at extension of link DF and EG. The aem FP and GQ are pivoted at P and Q respectvily

Consider the equilibrium forces one-half of governor as shown in fig b.the instantneous centr I lies on on the intersection of line PF produced and from D drawn perpendicular to spindle axis. The perpendicular BM is drawn on ID

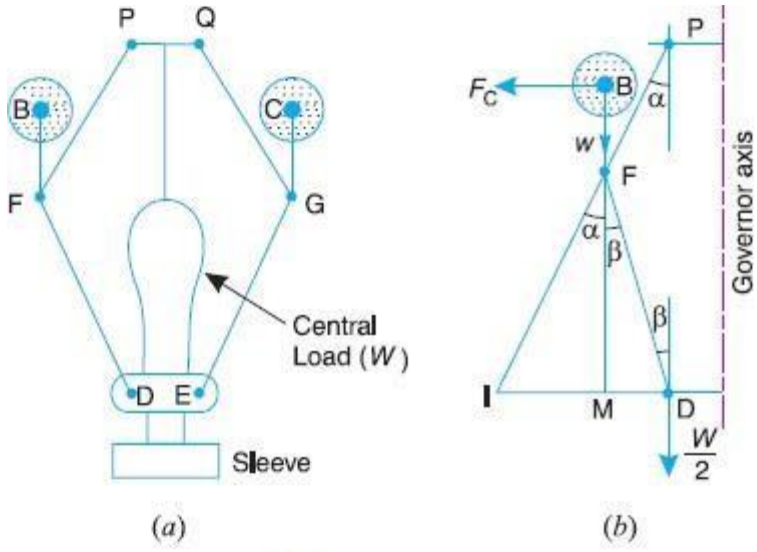


Fig. Proell governor.

Takingmomentsabout*I*,usingthesamenotations

$$F_C \times BM = w \times IM + \frac{W}{2} \times ID = m.g \times IM + \frac{M.g}{2} \times ID \quad \ldots (i)$$

$$\therefore \quad F_C = m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM + MD}{BM}\right) \quad \ldots (\because \; ID = IM + MD)$$

Multiplying and dividing by FM, we have

$$F_C = \frac{FM}{BM}\left[m.g \times \frac{IM}{FM} + \frac{M.g}{2}\left(\frac{IM}{FM} + \frac{MD}{FM}\right)\right]$$

$$= \frac{FM}{BM}\left[m.g \times \tan\alpha + \frac{M.g}{2}(\tan\alpha + \tan\beta)\right]$$

$$= \frac{FM}{BM} \times \tan\alpha\left[m.g + \frac{M.g}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right)\right]$$

We know that $F_C = m.\omega^2 r$; $\tan\alpha = \frac{r}{h}$ and $q = \frac{\tan\beta}{\tan\alpha}$

$$\therefore \quad m.\omega^2.r = \frac{FM}{BM} \times \frac{r}{h}\left[m.g + \frac{M.g}{2}(1+q)\right]$$

and

$$\omega^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{g}{h} \quad \ldots (ii)$$

Substituting $\omega = 2\pi N/60$, and $g = 9.81$ m/s², we get

$$N^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{895}{h} \quad \ldots (iii)$$

HartnellGovernor

A Hartnell governor is a spring loaded governor .It consists of two bell crank levers pivoted at thepoints *O*,*O* to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm *OB* and a roller at the end of the horizontal arm *OR*. A helical spring in compression provides equal downward forces on the tworollers through acollar on the sleeve.The spring force may be adjusted by screwing a nut up or downon the sleeve.

Let m = Mass of each ball in kg,

M = Mass of sleeve in kg,

r_1 = Minimum radius of rotation in metres,

r_2 = Maximum radius of rotation in metres

ω_1 = Angular speed of the governor at minimum radius in rad/s,

ω_2 = Angular speed of the governor at maximum radius in rad/s,

S_1 = Spring force exerted on the sleeve S_2 = Spring force exerted on the sleeve

at F_{C1} = Centrifugal force = $m (\omega_1)^2 r$

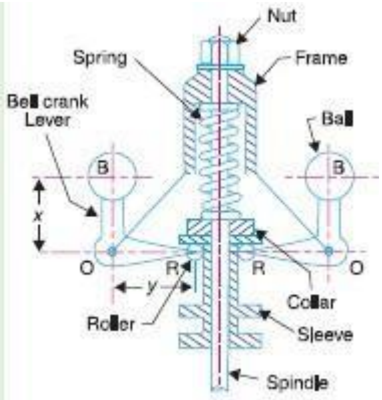
F_{C2} = Centrifugal force at = $m(\omega_2)^2 r_2$,

s = Stiffness of the spring or the force required to compress the spring by one mm,

x = Length of the vertical or ball arm of the lever in metres,

y = Length of the horizontal or sleeve arm of the lever in metres, and

r = Distance of fulcrum O from the governor axis or the radius of rotation when the governor is in mid- position.



Consider the forces acting at one bell crank lever. The minimum and maximum position is shown in Fig.. Let h be the compression of the spring when the radius of rotation changes from r_1 to r_2.

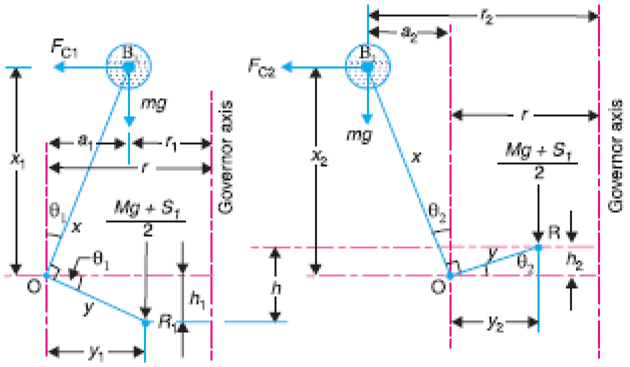
$$\frac{h_1}{y} = \frac{a_1}{x} = \frac{r - r_1}{x} \quad \ldots (i)$$

$$\frac{h_2}{y} = \frac{a_2}{x} = \frac{r_2 - r}{x} \quad \ldots (ii)$$

Adding equations (*i*) and (*ii*),

$$\frac{h_1 + h_2}{y} = \frac{r_2 - r_1}{x} \quad \text{or} \quad \frac{h}{y} = \frac{r_2 - r_1}{x} \quad \ldots (\because h = h_1 + h_2)$$

$$\therefore \quad h = (r_2 - r_1)\frac{y}{x} \quad \ldots (iii)$$



(*a*) Minimum position. (*b*) Maximum position.

Now for minimum position, taking moments about point O, we get

$$\frac{M.g + S_1}{2} \times y_1 = F_{C1} \times x_1 - m.g \times a_1$$

or

$$M.g + S_1 = \frac{2}{y_1}(F_{C1} \times x_1 - m.g \times a_1) \quad \ldots (iv)$$

y_1

Again for maximum position, taking moments about point O, we get

$$\frac{M.g + S_2}{2} \times y_2 = F_{C2} \times x_2 + m.g \times a_2$$

or

$$M.g + S_2 = \frac{2}{y_2}(F_{C2} \times x_2 + m.g \times a_2) \quad \ldots (v)$$

Subtracting equation (*iv*) from equation (*v*),

$$S_2 - S_1 = \frac{2}{y_2}(F_{C2} \times x_2 + m.g \times a_2) - \frac{2}{y_1}(F_{C1} \times x_1 - m.g \times a_1)$$

We know that

$$S_2 - S_1 = h.s, \quad \text{and} \quad h = (r_2 - r_1)\frac{y}{x}$$

$$\therefore \quad s = \frac{S_2 - S_1}{h} = \left(\frac{S_2 - S_1}{r_2 - r_1}\right)\frac{x}{y}$$

Neglecting the obliquity effect of the arms (*i.e.* $x_1 = x_2 = x$, and $y_1 = y_2 = y$) and the moment due to weight of the balls (*i.e.* $m.g$), we have for minimum position,

$$\frac{M.g + S_1}{2} \times y = F_{C1} \times x \quad \text{or} \quad M.g + S_1 = 2F_{C1} \times \frac{x}{y} \quad \ldots (vi)$$

Similarly for maximum position,

$$\frac{M.g + S_2}{2} \times y = F_{C2} \times x \quad \text{or} \quad M.g + S_2 = 2F_{C2} \times \frac{x}{y} \quad \ldots (vii)$$

Subtracting equation (*vi*) from equation (*vii*),

$$S_2 - S_1 = 2(F_{C2} - F_{C1})\frac{x}{y} \quad \ldots (viii)$$

We know that

$$S_2 - S_1 = h.s, \quad \text{and} \quad h = (r_2 - r_1)\frac{y}{x}$$

$$\therefore \quad s = \frac{S_2 - S_1}{h} = 2\left(\frac{F_{C2} - F_{C1}}{r_2 - r_1}\right)\left(\frac{x}{y}\right)^2 \quad \ldots (ix)$$

SensitivenessofGovernors

AgovernorissaidtobesensitiveifitschangeofspeedsfromnoLoadtofullloadmay beassmallafractionofthemeanequilibriumspeedaspossibleandthecorresponding sleeve lift may be as large as possible.

Suppose$\omega 1$=max.Equilibriumspeed $\omega 2$

= min. equilibrium speed

ω=meanequilibriumspeed=$(\omega 1+\omega 2)/2$

Therefore sensitiveness = $(\omega 1- \omega 2)/2$

Stability of Governors

A governor is said to be ***stable*** when for every speed within the working range there is definite configuration *i.e.* there is only one radius of rotation of the governor balls at which the governor equilibrium. For a stable governor, if the equilibrium speedincreases, the radius of governor balls must also increase.

IsochronousGovernors

This is an extreme case of sensitiveness. When the equilibrium speed is constant for all radii of rotation of the balls within the working range, the governor is said to be in isochronism. This means that the difference between the maximum and minimum equilibrium speeds is zero and the sensitiveness shall be infinite.

FLYWHEEL:-

Aflywheelisawheelofheavymassmountedonthecrankshaftanditstoresenergyduringtheperiod when the supply of energy is more

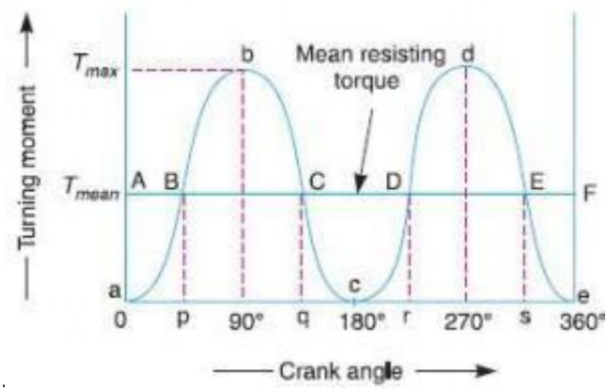
duringtheperiod whentheflywheelabsorbsenergyitsspeedincreasesandduring the period when it releases energy its speed decreases.

Inengine, the flywheelabsorbs the stroke and givesout the energyduring idle strokes andthus keeps themaximumspeedandminimumspeedofcrankshaftnearthe meanshaftinathermodynamiccycle. Inpower press, the flywheel absorbs the mechanical energy produced by electric motor during idle period and givesthe energy when actual operation is performed.Inthis waywith the use of flywheel, motor of smaller capacity is able to serve thepurpose.

FluctuationofEnergy

Thefluctuationofenergymaybedeterminedbytheturningmomentdiagramforonecompletecycleof operation. Consider the turning moment diagram for a single cylinder double acting steam engineasshownin Fig.

Wesee that the meanresisting torque line *AF* cuts the turning moment diagram at points *B*, *C*, *D* and *E*. When the crank moves from *a* to *p*, the work done by the engine is equal to the area*aBp*, whereasthe energy required is represented by the area*aABp*. Inother words, the engine has done less work (equal to the area *a AB*) than the requirement. This amount of energy istaken fromtheflywheelandhence thespeedoftheflywheeldecreases. Nowthecrank moves from *p* to *q*, the work done by the engine is equal to the area *pBbCq*, whereas the requirement of energy is represented by the area *pBCq*. Therefore, the engine has done more work than therequirement.

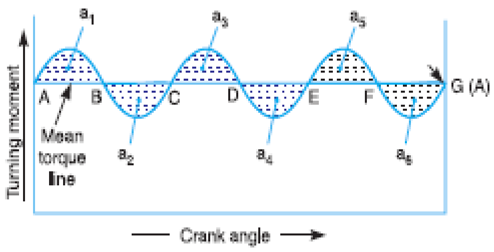


Thisexcess work(equal to thearea *BbC*) isstoredin the flywheel andhence the speedof the flywheel increases while the crank movesfrom *p* to *q*. Similarly, whenthe crank moves from *q* to *r*, morework is taken from the engine than is developed. This loss of work is represented by the area *Cc D*. To supply this loss, the flywheel gives up someof its energy and thus the speed decreases while the crank moves from *q* to *r*. Asthe crank movesfrom *r* to *s*, excess energy is again developed given by the area *D d E* and the speed again increases. Asthe piston movesfrom *s* to *e*, again there is a loss of work and the speed decreases. The variations of energy above and below the meanresisting torque line are called ***fluctuations of energy***.

Maximumfluctuationofenergy

DeterminationofMaximumFluctuationofEnergy

Aturningmomentdiagramforamulti-cylinderengineisshownbyawavycurveinFig.



Thehorizontal line *AG* represents the mean torque line. Leta_1, a_3, $a_5$be the areas above the meantorquelineanda_2,a_4 and$a_6$betheareasbelowthemeantorqueline.Theseareasrepresent somequantityofenergywhichiseitheraddedorsubtractedfromtheenergyofthemovingpartsof the engine.

Lettheenergyintheflywheelat A=E, then

from Fig. 16.4, we have

Energyat $B=E+a_1$

Energyat$C=E+a_1-a_2$

Energyat $D=E+a_1-a_2+a_3$

Energyat $E=E+a_1-a_2+a_3-a_4$

Energyat $F=E+a_1-a_2+a_3-a_4+a_5$

Energy at $G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$= Energy at A (*i.e.* cyclerepeats after G)

Letusnowsupposethatthegreatestoftheseenergiesisat Bandleastat$E$.Therefore,

Maximum energy in flywheel= $E + a_1$

Minimumenergyintheflywheel=$E+a_1-a_2+a_3-a_4$

ΔE=Maximumfluctuationofenergy,

ΔE=Maximumenergy–Minimumenergy

$=(E + a_1) - (E + a_1 - a_2 + a_3 - a_4) = a_2 - a_3 + a_4$

CoefficientofFluctuationofEnergy

Itmaybedefinedasthe **ratioofthemaximumfluctuationofenergytotheworkdone per cycle.**

Mathematically, coefficient of fluctuation of energy

C_E=Maximumfluctuationofenergy/Workdonepercycle

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

Fluctuationofspeed

Whenflywheelabsorbsenergyitsspeedrisesandwhenit givesupenergyitsspeeddecreases.The difference between maximum speed and minimum speed is called as fluctuation of speed.

CoefficientofFluctuationofspeed

Theratiooffluctuationofspeedtothemeanspeediscalled CoefficientofFluctuationof speed It is denoted as K_s.

ω_1=maximumangularvelocityofflywheelatmaximumspeedinr.p.mN_1

ω_2=minimumangularvelocityofflywheelatminimumspeed in r.p.mN_2

ω=mean angular velocity of flywheel at mean speed in r.p.mN

According to the theory,

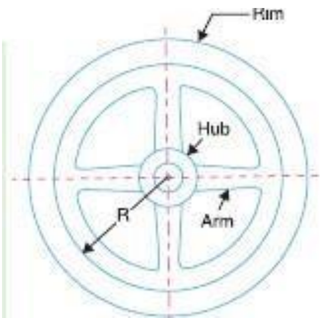
$$K_s = (\omega_1 - \omega_2)/\omega$$

EnergyStoredinaFlywheel

A flywheel is shown in Fig. thatwhenaflywheelabsorbs energy,its speedincreases and when it gives up energy,its speeddecreases.

Let$m$=Massoftheflywheelinkg,

k=Radiusofgyrationoftheflywheelinmetres



I = Mass moment of inertia of the flywheel about its axis of rotation in kg-m^2 = $m.k^2$,

N_1 and N_2 = Maximum and minimum speeds during the cycle in r.p.m.,

ω_1 and ω_2 = Maximum and minimum angular speeds during the cycle in rad/s,

N = Mean speed during the cycle in r.p.m. $= \dfrac{N_1 + N_2}{2}$,

ω = Mean angular speed during the cycle in rad/s $= \dfrac{\omega_1 + \omega_2}{2}$,

C_S = Coefficient of fluctuation of speed, $= \dfrac{N_1 - N_2}{N}$ or $\dfrac{\omega_1 - \omega_2}{\omega}$

We know that the mean kinetic energy of the flywheel,

$$E = \frac{1}{2} \times I.\omega^2 = \frac{1}{2} \times m.k^2.\omega^2 \qquad \text{(in N-m or joules)}$$

As the speed of the flywheel changes from ω_1 to ω_2, the maximum fluctuation of energy,

$$\Delta E = \text{Maximum K.E.} - \text{Minimum K.E.}$$

$$= \frac{1}{2} \times I\,(\omega_1)^2 - \frac{1}{2} \times I\,(\omega_2)^2 = \frac{1}{2} \times I\left[(\omega_1)^2 - (\omega_2)^2\right]$$

$$= \frac{1}{2} \times I\,(\omega_1 + \omega_2)(\omega_1 - \omega_2) = I.\omega(\omega_1 - \omega_2) \qquad \text{...}(i)$$

$$\text{...}\left(\because \omega = \frac{\omega_1 + \omega_2}{2}\right)$$

$$= I.\omega^2\left(\frac{\omega_1 - \omega_2}{\omega}\right) \qquad \text{... (Multiplying and dividing by } \omega)$$

$$= I.\omega^2.C_S = m.k^2.\omega^2.C_S \qquad \text{...} (\because I = m.k^2) \quad \text{...}(ii)$$

$$= 2.E.C_S \text{ (in N–m or joules)} \qquad \text{...}\left(\because E = \frac{1}{2} \times I.\omega^2\right) \text{...}(iii)$$

Module-5

Explaintheconceptofbalancing:
Explainstaticbalancingofrotatingparts:
Explaintheprincipleofbalancingofreciprocatingmasses: State the causes and effect of unbalance:
Differentbetweenstaticanddynamicbalancing.:

Explaintheconceptofbalancing:

Balancing is the process of eliminating or at least reducing the ground forces and/or moments.Itisachievedbychangingthelocationofthemasscentresoflinks. Balancingofrotatingpartsisawellknownproblem.Arotatingbodywithfixed rotationaxiscanbefullybalancedi.e.alltheinertiaforcesandmoments.For mechanism containing links rotating about axis which are not fixed, force balancing is possible, moment balancing by itself may be possible, but both not possible.Wegenerally try to do force balancing. A fully force balance is possible, but any action in force balancing severe themoment balancing

Balancingofrotatingmasses: Theprocessofprovidingthesecondmassinorderto counteracttheeffectof thecentrifugalforceof the firstmassis calledbalancingof rotating masses.

Static balancing: Thenetdynamicforceactingon theshaft is equalto zero.This requires thatthe lineofactionof threecentrifugalforcesmustbe thesame.Inother words,thecentreofthemassesofthesystemmustlieontheaxisoftherotation.This is the condition for static balancing.

Dynamicbalancing:Thenetcoupleduetodynamicforcesactingontheshaftis equaltozero.Thealgebraicsumof themomentsaboutanypointintheplanemustbe zero.

Staticbalancingofrotatingmass

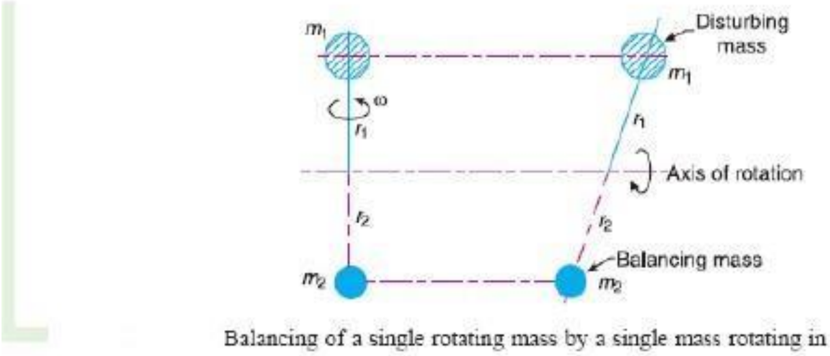
Balancingofasinglerotatingmassbysinglemassrotatinginthesameplane:

Consider a disturbing mass m_1 attached to a shaft rotating at ω rad/s as shown in Fig. Let r_1 be the radius of rotation of the mass m_1 (*i.e.* distance between the axis of rotation of the shaft and the centre of gravity of the mass m_1).

We know that the centrifugal force exerted by the mass m_1 on the shaft,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1 \quad \ldots (i)$$

This centrifugal force acts radially outwards and thus produces bending moment on the shaft. In order to counteract the effect of this force, a balancing mass (m_2) may be attached in the same plane of rotation as that of disturbing mass (m_1) such that the centrifugal forces due to the two masses are equal and opposite.



Balancing of a single rotating mass by a single mass rotating in the same plane.

Let r_2 = Radius of rotation of the balancing mass m_2 (*i.e.* distance between the axis of rotation of the shaft and the centre of gravity of mass m_2).

$\therefore$ Centrifugal force due to mass m_2,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2 \quad \ldots (ii)$$

Equating equations (*i*) and (*ii*),

$$m_1 \cdot \omega^2 \cdot r_1 = m_2 \cdot \omega^2 \cdot r_2 \quad \text{or} \quad m_1 \cdot r_1 = m_2 \cdot r_2$$

CASE2:

BALANCING OF ASINGLE ROTATINGMASS BY TWO MASSES ROTATING INDIFFERENTPLANES.

Therearetwopossibilitieswhileattachingtwobalancingmasses:

1. Theplaneofthedisturbing massmay be in between theplanesof the two balancingmasses.

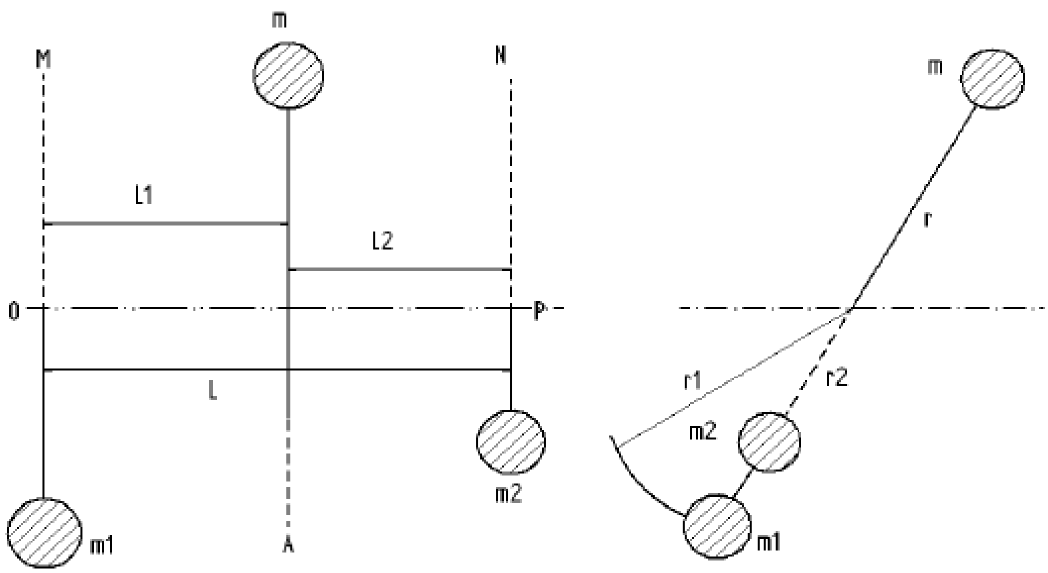
2. Theplaneofthedisturbingmassmaybeontheleftorrightsideoftwo planes containingthebalancingmasses.

Inordertobalanceasinglerotatingmassbytwomasses rotatingindifferent planes

whichareparalleltotheplaneofrotationofthedisturbingmassi)thenet dynamic force
acting on the shaft must be equal to zero, i.e. the centre of the masses of the system must lie on the axis of rotation and this is the condition for static balancingii)thenetcoupleduetothedynamicforcesactingontheshaftmust be equal to zero, i.e. the algebraic sum of the moments about any point in the plane must be zero. The conditions i) and ii) together give dynamic balancing.

THE PLANE OF THE DISTURBING MASS LIES IN BETWEEN THE PLANES OFTHETWOBALANCINGMASSES.

The plane of the disturbing mass lies inbetween the planes of the two balancing masses



Consider the disturbing mass m lying in a plane A which is to be balanced by two rotating masses m_1 and m_2 lying in two different planes M and N which are parallel to the plane A as shown.

Let r, r_1 and r_2 be the radii of rotation of the masses in planes A, M and N respectively.
Let L_1, L_2 and L be the distance between A and M, A and N, and M and N respectively.
Now,
The centrifugal force exerted by the mass m in plane A will be,

$$F_c = m\ \omega^2 r \text{ ------------------(1)}$$

Similarly,
The centrifugal force exerted by the mass m_1 in plane M will be,

$$F_{c1} = m_1 \omega^2 r_1 \text{------------------(2)}$$

And the centrifugal force exerted by the mass m_2 in plane N will be,

$$F_{c2} = m_2 \omega^2 r_2 ------------------(3)$$

For the condition of static balancing,

$$F_c = F_{c1} + F_{c2}$$
$$\text{or} \quad m\omega^2 r = m_1 \omega^2 r_1 + m_2 \omega^2 r_2$$
$$\text{i.e.} \quad m r = m_1 r_1 + m_2 r_2 ----------------(4)$$

Now, to determine the magnitude of balancing force in the plane 'M' or the dynamic force at the bearing 'O' of a shaft, take moments about ' P ' which is the point of intersection of the plane N and the axis of rotation.

Therefore,

$$F_{c1} x L = F_c x L_2$$
$$\text{or} \quad m_1 \omega^2 r_1 \, x \, L = m\omega^2 r \, x L_2$$
Therefore,
$$m_1 r_1 L = mrL_2 \quad \text{or} \quad m_1 r_1 = mr\frac{L_2}{L} ---------(5)$$

Similarly, in order to find the balancing force in plane 'N' or the dynamic force at the bearing 'P' of a shaft, take moments about ' O ' which is the point of intersection of the plane M and the axis of rotation.

Therefore,

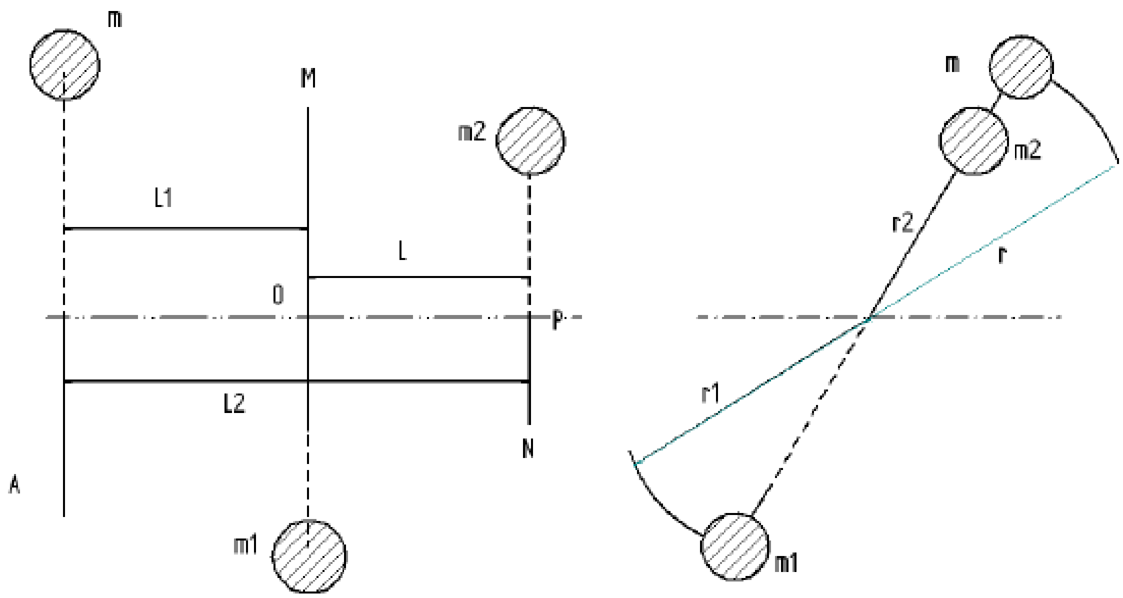
$$F_{c2} x L = F_c x L_1$$
$$\text{or} \quad m_2 \omega^2 r_2 \, x \, L = m\omega^2 r \, x L_1$$
Therefore,
$$m_2 r_2 L = mrL_1 \quad \text{or} \quad m_2 r_2 = mr\frac{L_1}{L} ---------(6)$$

For dynamic balancing equations (5) or (6) must be satisfied along with equation (4).

CASE 2(II):

WHEN THE PLANE OF THE DISTURBING MASS LIES ON ONE END OF THE TWO PLANES CONTAINING THE BALANCING MASSES.

When the plane of the disturbing mass lies on one end of the planes of the balancing masses



For static balancing,

$$F_{c1} = F_c + F_{c2}$$

$$\text{or} \quad m_1 \omega^2 r_1 = m \omega^2 r + m_2 \omega^2 r_2$$

$$\text{i.e.} \quad m_1 r_1 = m r + m_2 r_2 \text{-----------------}(1)$$

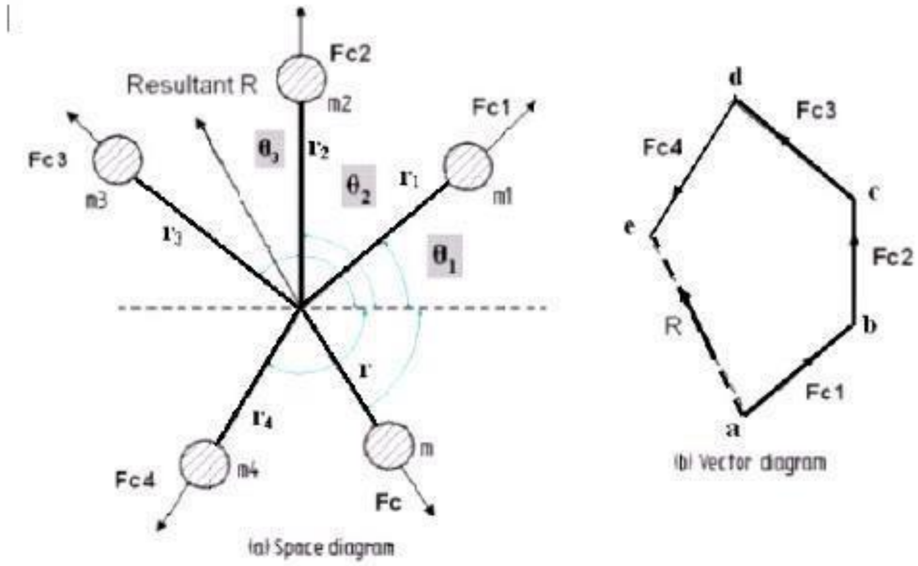
For dynamic balance the net dynamic force acting on the shaft and the net couple due to dynamic forces acting on the shaft is equal to zero.
To find the balancing force in the plane 'M' or the dynamic force at the bearing 'O' of a shaft, take moments about 'P'. i.e.

$$F_{c1} \times L = F_c \times L_2$$
$$\text{or} \quad m_1 \omega^2 r_1 \times L = m\omega^2 r \times L_2$$
Therefore,
$$m_1 r_1 L = mrL_2 \quad \text{or} \quad m_1 r_1 = mr\frac{L_2}{L} \text{---------}(2)$$

Similarly, to find the balancing force in the plane 'N' , take moments about 'O', i.e.,

$$F_{c2} \times L = F_c \times L_1$$
$$\text{or} \quad m_2 \omega^2 r_2 \times L = m\omega^2 r \times L_1$$
Therefore,
$$m_2 r_2 L = mrL_1 \quad \text{or} \quad m_2 r_2 = mr\frac{L_1}{L} \text{---------}(3)$$

CASE 3:
BALANCING OF SEVERAL MASSES ROTATING IN THE SAME PLANE



BALANCING OF SEVERAL MASSES ROTATING IN THE SAME PLANE

Consider a rigid rotor revolving with a constant angular velocity _ rad/s. A number of masses say, four are depicted by point masses at different radii in the same transverse plane

If m_1, m_2, m_3 and m_4 are the masses revolving at radii r_1, r_2, r_3 and r_4 respectively in the same plane.
The centrifugal forces exerted by each of the masses are F_{c1}, F_{c2}, F_{c3} and F_{c4} respectively.
Let F be the vector sum of these forces. i.e.

$$F = F_{c1} + F_{c2} + F_{c3} + F_{c4}$$
$$= m_1 \omega^2 r_1 + m_2 \omega^2 r_2 + m_3 \omega^2 r_3 + m_4 \omega^2 r_4 ---------- (1)$$

The rotor is said to be statically balanced if the vector sum F is zero. If the vector sum F is not zero, i.e. the rotor is unbalanced, then introduce a counterweight (balance weight) of mass 'm' at radius 'r' to balance the rotor so that,

$$m_1 \omega^2 r_1 + m_2 \omega^2 r_2 + m_3 \omega^2 r_3 + m_4 \omega^2 r_4 + m \omega^2 r = 0 ---------- (2)$$

or

$$m_1 r_1 + m_2 r_2 + m_3 r_3 + m_4 r_4 + m r = 0 ----------------- (3)$$

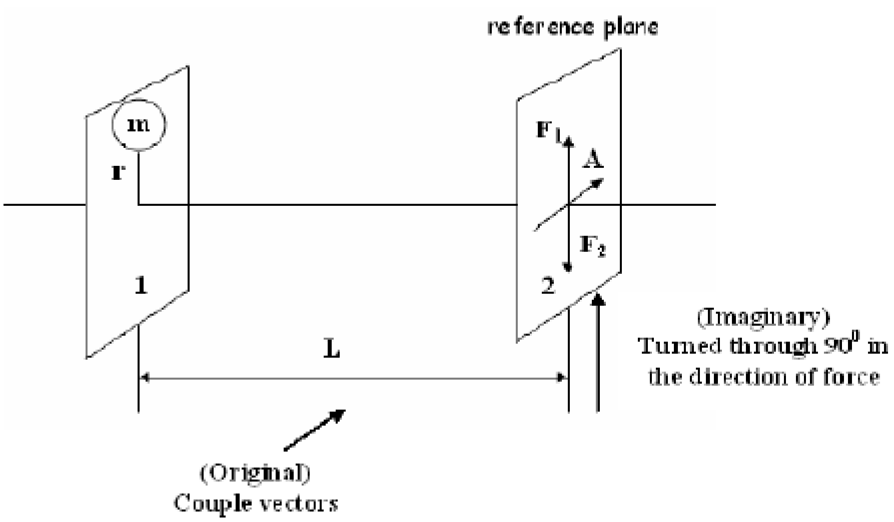
The magnitude of either 'm' or 'r' may be selected and the other can be calculated.
In general, if $\sum \mathbf{m_i r_i}$ is the vector sum of $\mathbf{m_1 r_1}$, $\mathbf{m_2 r_2}$, $\mathbf{m_3 r_3}$, $\mathbf{m_4 r_4}$ etc, then,

$$\sum m_i r_i + mr = 0 --------- (4)$$

CASE 4:

BALANCING OF SEVERAL MASSES ROTATING IN DIFFERENT PLANES

When several masses revolve in different planes, they may be transferred to a reference plane and this reference plane is a plane passing through a point on the axis of rotation and perpendicular to it.



When a revolving mass in one plane is transferred to a reference plane, its effect is to causeaforceofsamemagnitudetothecentrifugalforceoftherevolvingmasstoactin the reference plane along with a couple of magnitude equal to the product of the force and the distance between the two planes.
Inordertohaveacompletebalanceoftheseveralrevolvingmassesindifferentplanes,

1. theforcesinthereferenceplanemustbalance,i.e.,theresultantforcemustbezeroand
2. thecouplesaboutthereferenceplanemustbalancei.e.,theresultantcouplemustbe zero.

A mass placed in the reference plane may satisfy the first condition but the couple balanceissatisfiedonlybytwoforcesofequalmagnitudeindifferentplanes.Thus,in general, two planes are needed to balance a system of rotating masses

balanicingofreciprocatingengine

SliderCrankMechanism:

PrimaryAndSecondaryAcceleratingForce

Acceleration of the reciprocating mass of a slider-crank mechanism is given by,

$$a_p = \text{Acceleration of piston}$$

$$= r\omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right] ---------(1)$$

$$\text{Where } n = \frac{l}{r}$$

And, the force required to accelerate the mass 'm' is

$$F_i = m r \omega^2\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

$$= m r \omega^2 \cos\theta + m r \omega^2 \frac{\cos 2\theta}{n} ---------(2)$$

The first term of the equation (2) , i.e. $m r \omega^2 \cos\theta$ is called **primary accelerating force** the second term $m r \omega^2 \frac{\cos 2\theta}{n}$ is called the **secondary accelerating force.**

Maximum value of primary accelerating force is $m r \omega^2$

And Maximum value of secondary accelerating force is $\frac{m r \omega^2}{n}$

Generally, 'n' value is much greater than one; the secondary force is small compared to primary force and can be safely neglected for slow speed engines.

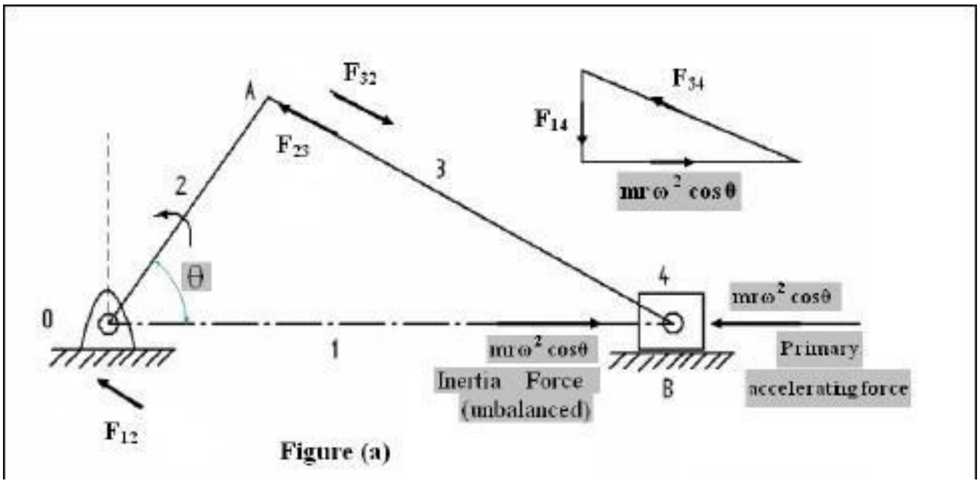


Figure (a)

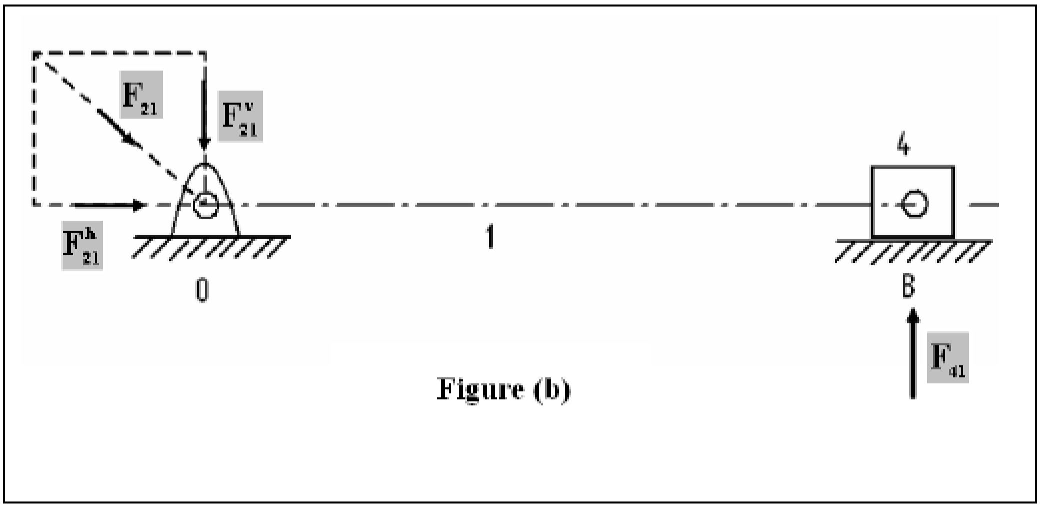


Figure (b)

In Fig (b), the forces acting on the engine frame due to inertia force are shown.

At 'O' the force exerted by the crankshaft on the main bearings has two components, horizontal $\mathbf{F}_{21}^{h}$ and vertical $\mathbf{F}_{21}^{v}$.

$\mathbf{F}_{21}^{h}$ is an horizontal force, which is an **unbalanced shaking force.**

$\mathbf{F}_{21}^{v}$ and $\mathbf{F}_{41}^{v}$ balance each other but form an **unbalanced shaking couple**.

The magnitude and direction of these unbalanced force and couple go on changing with angle θ. The shaking force produces linear vibrations of the frame in horizontal direction, whereas the shaking couple produces an oscillating vibration.

The shaking force $\mathbf{F}_{21}^{h}$ is the only unbalanced force which may hamper the smooth running of the engine and effort is made to balance the same.
However it is not at all possible to balance it completely and only some modifications can be carried out.

BalancingOfTheShakingForce:

Shaking force is being balanced by adding a rotating counter mass at radius 'r' directly opposite the crank. Thisprovides onlya partial balance. Thiscountermass isin addition to themassused to balance the rotating unbalance due to the mass at the crank pin. This is shown in figure (c).

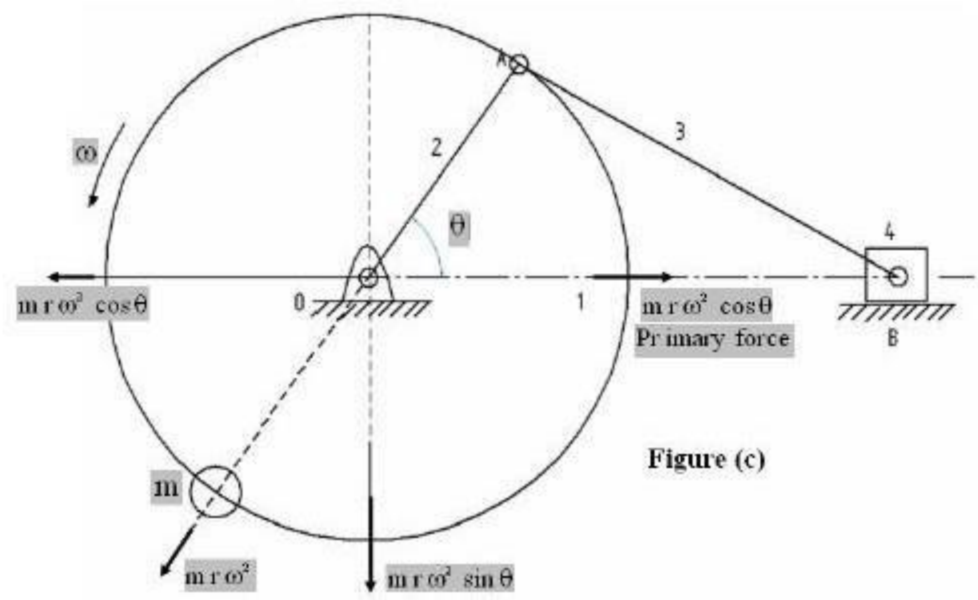


Figure (c)

The horizontal component of the centrifugal force due to the balancing mass is $\mathbf{m\,r\,\omega^2 \cos\theta}$ and this is in the line of stroke. This component neutralizes the unbalanced reciprocating force. But the rotating mass also has a component $\mathbf{m\,r\,\omega^2 \sin\theta}$ perpendicular to the line of stroke which remains unbalanced. The unbalanced force is zero at $\theta = 0^0$ or 180^0 and maximum at the middle of the stroke i.e. $\theta = 90^0$. The magnitude or the maximum value of the unbalanced force remains the same i.e. equal to $\mathbf{m\,r\,\omega^2}$. Thus instead of sliding to and fro on its mounting, the mechanism tends to jump up and down.

To minimize the effect of the unbalance force a compromise is, usually made, is $\frac{2}{3}$ of the reciprocating mass is balanced or a value between $\frac{1}{2}$ to $\frac{3}{4}$.

If 'c' is the fraction of the reciprocating mass, then

The primary force balanced by the mass = $\mathbf{c m r \omega^2 \cos\theta}$

and

The primary force unbalanced by the mass = $\mathbf{(1-c)\, m r \omega^2 \cos\theta}$

Vertical component of centrifugal force which remains unbalanced
$\mathbf{= c\ m r \omega^2 \sin\theta}$

In reciprocating engines, unbalance forces in the direction of the line of stroke are more dangerous than the forces perpendicular to the line of stroke.

Resultant unbalanced force at any instant

$$=\sqrt{[(1-c)mr\omega^2\cos\theta]^2+[cmr\omega^2\sin\theta]^2}$$

The resultant unbalanced force is minimum when, $c=\dfrac{1}{2}$

This method is just equivalent to as if a revolving mass at the crankpin is completely balanced by providing a counter mass at the same radius diametrically opposite to the crank. Thus if m_P is the mass at the crankpin and 'c' is the fraction of the reciprocating mass 'm' to be balanced , the mass at the crankpin may be considered as $cm+m_P$ which is to be completely balanced.

SECONDARY BALANCING:

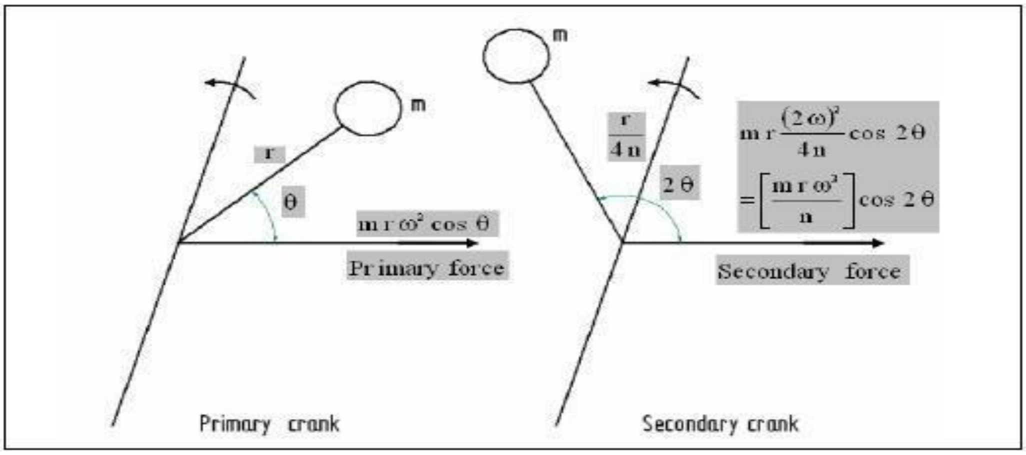
Secondary acceleration force is equal to $mr\omega^2\dfrac{\cos 2\theta}{n}$ -------(1)

Its frequency is twice that of the primary force and the magnitude $1/n$ times the magnitude of the primary force.

The secondary force is also equal to $mr(2\omega)^2\dfrac{\cos 2\theta}{4n}$ ------(2)

Consider, two cranks of an engine, one actual one and the other imaginary with the following specifications.

	Actual	Imaginary
Angular velocity	ω	2ω
Length of crank	r	$\dfrac{r}{4n}$
Mass at the crank pin	m	m



Centrifugal force induced in the imaginary crank = $\frac{mr(2\omega)^2}{4n}$

Component of this force along the line of stroke is = $\frac{mr(2\omega)^2}{4n}\cos 2\theta$

Thus the effect of the secondary force is equivalent to an imaginary crank of length $\frac{r}{4n}$ rotating at double the angular velocity, i.e. twice of the engine speed. The imaginary crank coincides with the actual at inner top-dead centre. At other times, it makes an angle with the line of stroke equal to twice that of the engine crank.

The secondary couple about a reference plane is given by the multiplication of the secondary force with the distance ‘ 1’ of the plane from the reference plane.

COMPLETE BALANCING OF RECIPROCATING PARTS

Conditions to be fulfilled:

1. Primary forces must balance i.e., primary force polygon is enclosed.
2. Primary couples must balance i.e., primary couple polygon is enclosed.
3. Secondary forces must balance i.e., secondary force polygon is enclosed.
4. Secondary couples must balance i.e., secondary couple polygon is enclosed.

Usually, it is not possible to satisfy all the above conditions fully for multi-cylinder engine. Mostly some unbalanced force or couple would exist in the reciprocating engines.

Module-6

Vibration of machine parts 6.1 Introduction to Vibration and related terms (Amplitude,time period and frequency, cycle) 6.2 Classification of vibration. 6.3 Basic concept of natural,forced&dampedvibration6.4TorsionalandLongitudinalvibration.6.5 Causes & remedies of vibration

Introduction

Whenelastic bodiessuchasaspring,abeamandashaftaredisplacedfrom theequilibrium position by the application of external forces, and then released, they execute a***vibratorymotion*****.**

Classifyvibrations

1- Freeornaturalvibrations

- Longitudinalvibrations,
- Transversevibrations
- Torsionalvibrations.

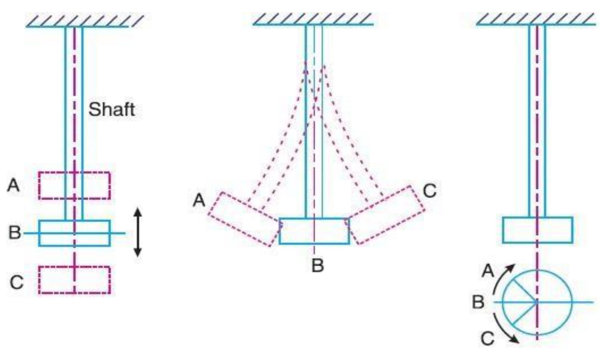
2- Forcedvibrations

3- Dampedvibrations**.**

NaturalVibration

Whennoexternalforceactsonthebody,aftergivingitaninitialdisplacement,thenthebodyis said to be under ***free or natural vibrations.*** The frequency of the free vibrations is called ***free or natural frequency.***

- ***Longitudinalvibrations***.Whentheparticlesoftheshaftordiscmovesparalleltothe axisof the shaft,asshownin Fig (*a*), thenthe vibrations areknownas***longitudinal vibrations*****.** Inthiscase,theshaftiselongatedandshortened alternatelyandthusthetensileand compressive stressesareinducedalternatelyintheshaft.Purelylongitudinalvibrationoccurswhenall particles of the bodymovein only one direction.



B = Mean position ; A and C = Extreme positions.
(*a*) Longitudinal vibrations. (*b*) Transverse vibrations. (*c*) Torsional vibrations.

- **2.** ***Transverse vibrations***. When the particles of the shaft or disc move approximatelyperpendiculartotheaxisoftheshaft,asshowninFig.(*b*),thenthe vibrationsareknownas***transversevibrations.***Inthiscase,theshaftisstraightand bent alternately and bending stresses are induced in the shaft.

- **3.*Torsionalvibrations***.Whentheparticlesoftheshaftordiscmoveinacircleabout the axis of the shaft, as shown in Fig.(*c*), then the vibrations are known as ***torsional vibrations.***Inthiscase,theshaftistwistedanduntwistedalternatelyandthetorsional shear stresses are inducedin the shaft.

2- Forcedvibrations.

When the body vibrates under the influence of external force, then the body is said to be under ***forced vibrations.*** The external force applied to the body is a periodic disturbing force created by unbalance. The vibrations have the same frequency as the applied force.

3- Dampedvibrations.

Whenthereisareductioninamplitudeovereverycycleofvibration,themotionis saidtobe ***damped vibration.*** This is due to the fact that a certain amount of energy possessed by the vibrating system is always dissipated in overcoming frictional resistances to the motion.

Definewithrespecttovibration

Cycle:

Amplitude:

TimePeriod:

1. *Periodofvibrationortimeperiod.*Itisthetimeintervalafterwhichthemotionisrepeated itself. The period of vibration is usually expressed inseconds.

2. *Cycle.*Itisthemotioncompletedduringonetimeperiod.

3. *Frequency.*Itisthenumberofcyclesdescribedinonesecond.InS.I.units,thefrequency is expressedin hertz (briefly written as Hz) which is equal to one cycle per second.

StatethecausesofVibration

Unbalance: This is basically in reference to the rotating bodies. The uneven distribution of mass in a rotating body contributes to the unbalance. A good example of unbalance related vibration would be the ―vibrating alert‖ in our mobile phones. Here a small amount of unbalancedweight is rotated by amotor causingthevibrationwhich makesthe mobile phone to vibrate. You would have experienced the samesort of vibration occurringinyourfrontloadedwashingmachinesthattendtovibrateduringthe ―spinning‖mode.

Misalignment: This is an other major cause of vibration particularly in machines that are driven by motors or any other prime movers.

Bent Shaft: A rotating shaft that is bent also produces the the vibrating effect since it losses it rotation capability about its center.

Gears in the machine: The gears in the machine always tend to produce vibration,mainly due to their meshing. Though this may be controlled to some extent, any problem in the gearbox tends to get enhanced withease.

Bearings: Last but not the least, here is a major contributor for vibration. In majority of the cases every initial problem starts in the bearings and propagates to the rest of the members of the machine. A bearing devoid of lubrication tends to wear out fast and fails quickly, but before this is noticed it damages the remaining components in the machine andan initial look would seem as if something hadgone wrong with the other components leading to the bearing failure.

Effectsof vibration:

(a) Bad Effects: The presence of vibration in any mechanical system produces unwanted noise, high stresses, poor reliability, wear and premature failure of parts. Vibrations are a great source of human discomfort in the form of physical and mentalstrains.

(b) GoodEffects: Avibrationdoesusefulworkinmusicalinstruments,vibratingscreens, shakers, relive pain in physiotherapy

Methodsofreductionofvibration:

- -unbalanceisits maincause,sobalancingofpartsisnecessary.
- usingshock absorbers.
- usingdynamicvibrationabsorbers.
- providingthescreens(ifnoiseisto bereduced)

Module-2

FrictionClutches

A frictionclutch has its principal applicationin the transmission ofpowerof shafts and machineswhichmustbestartedandstoppedfrequently.Itsapplicationisalso foundincases in which power is to be deliveredto machines partially or fully loaded. Theforce of friction is used to startthe drivenshaft from rest and gradually brings it up to the proper speed without excessiveslippingofthefrictionsurfaces.Inautomobiles,frictionclutchisusedtoconnectthe engine tothedrivenshaft.Inoperatingsuchaclutch,careshould betakenso thatthefriction surfaces engage easily and gradually brings the drivenshaft up to proper speed. Theproper alignmentofthebearingmustbemaintainedanditshouldbelocatedasclosetotheclutchas possible.

It may be noted that

1. Thecontactsurfacesshoulddevelopafrictionalforcethatmaypickupandholdtheload with reasonably low pressure between the contact surfaces.
2. Theheatoffrictionshouldberapidlydissipatedandtendencytograbshouldbeata minimum.
3. Thesurfacesshouldbebackedbyamaterialstiffenoughtoensureareasonablyuniform distribution of pressure.

Thefrictionclutchesofthefollowingtypesareimportantfromthesubjectpointofview:

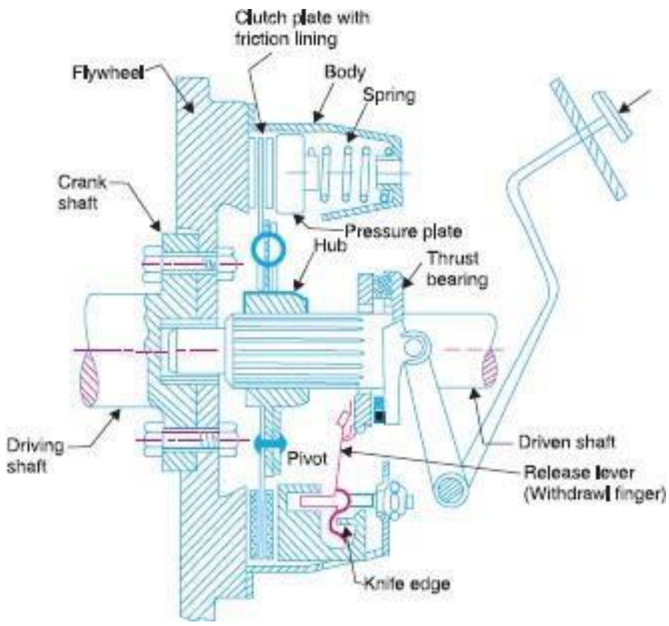
1. Discorplateclutches(singlediscormultiplediscclutch),
2. Coneclutches,and
3. Centrifugalclutches.

SingleDiscorPlateClutch

A single disc or plate clutch, as shownin Fig. 10.21, consists ofa clutch plate whoseboth sides arefaced witha friction material (usually of Ferrodo). It is mountedonthe hubwhich is freetomoveaxiallyalongthesplinesofthedrivenshaft.Thepressureplateismountedinside theclutchbodywhichisboltedtotheflywheel.Boththepressureplateandthe flywheelrotate withthe engine crankshaft or the driving shaft. Thepressure plate pushes the clutchplate towards the flywheelby a set ofstrong springs which arearrangedradially inside the body.

Thethreelevers (alsoknownas release levers orfingers) arecarried onpivots suspended fromthecaseofthebody.Thesearearrangedinsuchamannersothatthepressureplate moves away from the flywheel by the inward movement of a thrust bearing. The bearing is mounted upon a forked shaft and moves forward when the clutch pedal is pressed.

Whentheclutchpedalispresseddown,itslinkageforcesthethrustreleasebearingtomovein towards the flywheeland pressingthe longer ends oftheleversinward.Theleversareforced to turn ontheirsuspended pivotand the pressure plate movesaway fromthe flywheelby the knife edges, thereby compressing the clutch springs. This action removesthe pressure from the clutch plate and thus movesback fromthe flywheeland the drivenshaft becomes stationary.On the other hand, whenthe footis taken off from the clutch pedal, thethrust bearingmovesbackbythelevers.Thisallowsthespringstoextendandthusthepressure plate pushes the clutch plate back towards the flywheel.



Let T = Torque transmitted by the clutch,

p = Intensity of axial pressure with which the contact surfaces are h together,

r_1 and r_2 = External and internal radii of friction faces, and

μ = Coefficient of friction.

Consider an elementary ring of radius r and thickness dr as shown in Fig. 10.22 (*b*).

We know that area of contact surface or friction surface,

$$= 2\pi r.dr$$

∴ Normal or axial force on the ring,

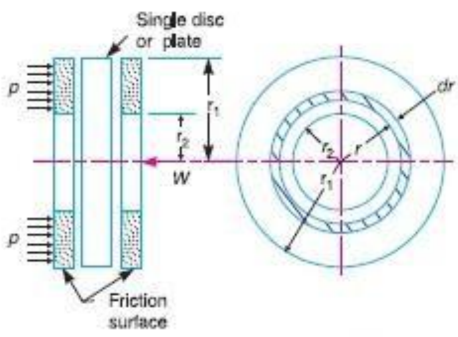
$$\delta W = \text{Pressure} \times \text{Area} = p \times 2\pi r.dr$$

and the frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W = \mu.p \times 2\pi r.dr$$

∴ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p \times 2\pi r.dr \times r = 2\pi \times \mu.p.r^2\, dr$$



(*a*) (*b*)

1. *Considering uniform pressure*

When the pressure is uniformly distributed over the entire area of the friction face, then the intensity of pressure,

$$p = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \quad \ldots(i)$$

where W = Axial thrust with which the contact or friction surfaces are held together.

We have discussed above that the frictional torque on the elementary ring of radius r and thickness dr is

$$T_r = 2\,\pi\,\mu.p.r^2\,dr$$

Integrating this equation within the limits from r_2 to r_1 for the total frictional torque.

∴ Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_1}^{r_2} 2\pi\mu.p.r^2.dr = 2\,\pi\mu\,p\left[\frac{r_3}{3}\right]_{r_2}^{r_1} = 2\,\pi\mu\,p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p from equation (i),

$$T = 2\,\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \frac{(r_1)^3 - (r_2)^3}{3}$$

$$= \frac{2}{3} \times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \mu.W.R$$

where R = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

2. *Considering uniform wear*

In Fig. 10.22, let p be the normal intensity of pressure at a distance r from the axis of the clutch. Since the intensity of pressure varies inversely with the distance, therefore

$$p.r. = C \text{ (a constant)} \quad \text{or} \quad p = C/r \quad \ldots(i)$$

and the normal force on the ring,

$$\delta W = p.2\pi r.dr = \frac{C}{r} \times 2\,\pi C.dr = 2\,\pi C.dr$$

∴ Total force acting on the friction surface,

$$W = \int_{r_2}^{r_1} 2\,\pi C\,dr = 2\,\pi C\left[r\right]_{r_2}^{r_1} = 2\,\pi C\,(r_1 - r_2)$$

or

$$C = \frac{W}{2\,\pi(r_1 - r_2)}$$

We know that the frictional torque acting on the ring,

$$T_r = 2\pi\mu.p\,r^2.dr = 2\pi\mu \times \frac{C}{r} \times r^2.dr = 2\pi\mu.C.r.dr$$

∴ Total frictional torque on the friction surface,

$$T = \int_{r_2}^{r_1} 2\pi\mu.C.r.dr = 2\,\pi\mu.C\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\,\pi\mu.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\mu.C[(r_1)^2 - (r_2)^2] = \pi\mu \times \frac{W}{2\,\pi(r_1 - r_2)}\left[(r_1)^2 - (r_2)^2\right]$$

$$= \frac{1}{2} \times \mu.W\,(r_1 + r_2) = \mu.W.R$$

where n = Number of pairs of friction or contact surfaces, and

R = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \quad \text{...(For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \quad \text{...(For uniform wear)}$$

2. For a single disc or plate clutch, normally both sides of the disc are effective. Therefore, a single disc clutch has two pairs of surfaces in contact, *i.e.* $n = 2$.

3. Since the intensity of pressure is maximum at the inner radius (r_2) of the friction or contact surface, therefore equation (*i*) may be written as

$$p_{max} \times r_2 = C \quad \text{or} \quad p_{max} = C/r_2$$

4. Since the intensity of pressure is minimum at the outer radius (r_1) of the friction or contact surface, therefore equation (*i*) may be written as

$$p_{min} \times r_1 = C \quad \text{or} \quad p_{min} = C/r_1$$

5. The average pressure (p_{av}) on the friction or contact surface is given by

$$p_{av} = \frac{\text{Total force on friction surface}}{\text{Cross-sectional area of friction surface}} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]}$$

6. In case of a new clutch, the intensity of pressure is approximately uniform but in an old clutch the uniform wear theory is more approximate.

7. The uniform pressure theory gives a higher frictional torque than the uniform wear theory. Therefore in case of friction clutches, uniform wear should be considered, unless otherwise stated.

Reference

TheoryOfMachineByRSKhurmiTheory

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